by the last rounding, that is, $.4403\times10^{-5}$, plus 3×10^{-6}, and so certainly cannot exceed $.8\times10^{-5}$.

(2) Lagrange's formula. In this example, the relevant formula is the 5-point one, given by

$$f=A_{-2}(p)f_{-2}+A_{-1}(p)f_{-1}+A_0(p)f_0+A_1(p)f_1+A_2(p)f_2$$

Tables of the coefficients $A_k(p)$ are given in chapter 25 for the range $p=0(.01)1$. We evaluate the formula for $p=.52$, .53 and .54 in turn. Again, in each evaluation we accumulate the $A_k(p)$ in the multiplier register since their sum is unity. We now have the following subtable.

x	$xe^xE_1(x)$		
7.952	.89772 9757		
		10622	
7.953	.89774 0379		−2
		10620	
7.954	.89775 0999		

The numbers in the third and fourth columns are the first and second differences of the values of $xe^xE_1(x)$ (see below); the smallness of the second difference provides a check on the three interpolations. The required value is now obtained by linear interpolation:

$$f_p=.3(.89772\ 9757)+.7(.89774\ 0379)=.89773\ 7192.$$

In cases where the correct order of the Lagrange polynomial is not known, one of the preliminary interpolations may have to be performed with polynomials of two or more different orders as a check on their adequacy.

(3) Aitken's method of iterative linear interpolation. The scheme for carrying out this process in the present example is as follows:

n	x_n	$y_n=xe^xE_1(x)$	$y_{0,n}$	$y_{0,1,n}$	$y_{0,1,2,n}$	$y_{0,1,2,3,n}$	x_n-x
0	8.0	.89823 7113					.0473
1	7.9	.89717 4302	.89773 44034				−.0527
2	8.1	.89927 7888	.89774 48264	.89773 71499			.1473
3	7.8	.89608 8737	2 90220	2394	.89773 71938		−.1527
4	8.2	.90029 7306	4 98773	1216	16	89773 71930	.2473
5	7.7	.89497 9666	2 35221	2706	43	30	−.2527

Here

$$y_{0,n}=\frac{1}{x_n-x_0}\begin{vmatrix}y_0 & x_0-x\\ y_n & x_n-x\end{vmatrix}$$

$$y_{0,1,n}=\frac{1}{x_n-x_1}\begin{vmatrix}y_{0,1} & x_1-x\\ y_{0,n} & x_n-x\end{vmatrix}$$

$$y_{0,1,\ldots,m-1,m,n}=\frac{1}{x_n-x_m}\begin{vmatrix}y_{0,1,\ldots,m-1,m} & x_m-x\\ y_{0,1,\ldots,m-1,n} & x_n-x\end{vmatrix}$$

If the quantities x_n-x and x_m-x are used as multipliers when forming the cross-product on a desk machine, their accumulation $(x_n-x)-(x_m-x)$ in the multiplier register is the divisor to be used at that stage. An extra decimal place is usually carried in the intermediate interpolates to safeguard against accumulation of rounding errors.

The order in which the tabular values are used is immaterial to some extent, but to achieve the maximum rate of convergence and at the same time minimize accumulation of rounding errors, we begin, as in this example, with the tabular argument nearest to the given argument, then take the nearest of the remaining tabular arguments, and so on.

The number of tabular values required to achieve a given precision emerges naturally in the course of the iterations. Thus in the present example six values were used, even though it was known in advance that five would suffice. The extra row confirms the convergence and provides a valuable check.

(4) Difference formulas. We use the central difference notation (chapter 25),

x_0	f_0				
		$\delta f_{1/2}$			
x_1	f_1		$\delta^2 f_1$		
		$\delta f_{3/2}$		$\delta^3 f_{3/2}$	
x_2	f_2		$\delta^2 f_2$		$\delta^4 f_2$
		$\delta f_{5/2}$		$\delta^3 f_{5/2}$	
x_3	f_3		$\delta^2 f_3$		
		$\delta f_{7/2}$			
x_4	f_4				

Here

$$\delta f_{1/2}=f_1-f_0,\ \delta f_{3/2}=f_2-f_1,\ \ldots,$$

$$\delta^2 f_1=\delta f_{3/2}-\delta f_{1/2}=f_2-2f_1+f_0$$

$$\delta^3 f_{3/2}=\delta^2 f_2-\delta^2 f_1=f_3-3f_2+3f_1-f_0$$

$$\delta^4 f_2=\delta^3 f_{5/2}-\delta^3 f_{3/2}=f_4-4f_3+6f_2-4f_1+f_0$$

and so on.

In the present example the relevant part of the difference table is as follows, the differences being written in units of the last decimal place of the function, as is customary. The smallness of the high differences provides a check on the function values

x	$xe^xE_1(x)$	$\delta^2 f$	$\delta^4 f$
7.9	.89717 4302	−2 2754	−34
8.0	.89823 7113	−2 2036	−39

Applying, for example, Everett's interpolation formula

$$f_p=(1-p)f_0+E_2(p)\delta^2 f_0+E_4(p)\delta^4 f_0+\ \ldots\ +pf_1+F_2(p)\delta^2 f_1+F_4(p)\delta^4 f_1+\ \ldots$$

and taking the numerical values of the interpolation coefficients $E_2(p)$, $E_4(p)$, $F_2(p)$ and $F_4(p)$ from Table 25.1, we find that