Approximating $Q(F|\nu_1,\nu_2)$

Example 23. Calculate $Q(1.714|10,40)$ using **26.6.15.**

The approximation given by **26.6.15** will result in a maximum error of .0005. For this example we have

$$x=\frac{(1.714)^{1/3}\left(1-\frac{2}{9(40)}\right)-\left(1-\frac{2}{9(10)}\right)}{\left[\frac{2}{9(10)}+(1.714)^{2/3}\frac{2}{9(40)}\right]^{\frac{1}{2}}}=1.2222$$

Interpolating in **Table 26.1** results in

$$Q(1.714|10,40)\approx Q(1.2222)=1-P(1.2222)=.1108$$

The correct value to 5D is $Q(1.714|10,40)=.11108$.

On the other hand the approximation given by **26.6.14** which is usually less accurate results in

$$x=\frac{\sqrt{[2(40)-1]\left(\frac{10}{40}\right)(1.714)}-\sqrt{2(10)-1}}{\sqrt{1+\frac{10}{40}(1.714)}}=1.2210$$

and interpolating in **Table 26.1** gives

$$Q(1.714|10,40)\approx Q(1.2210)=1-P(1.2210)=.1112$$

Calculation of F Outside the Range of Table 26.9

Example 24. Find the value of F for which $Q(F|10,20)\approx .0001$ using **26.6.16** and **26.5.22.**

For this problem we have $a=\frac{\nu_2}{2}=10$, $b=\frac{\nu_1}{2}=5$, $p=.0001$. The value of the normal deviate which cuts off .0001 in the tail of the distribution is $y=3.7190$ (i.e., $Q(3.7190)=.0001$). Hence substituting in **26.5.22** gives

$$h=2\left[\frac{1}{19}+\frac{1}{9}\right]^{-1}=12.2143$$

$$\lambda=\frac{3.7190^2-3}{6}=1.8052$$

$$w=3.7190\,\frac{(12.2143+1.8052)^{\frac{1}{2}}}{12.2143}-\left(\frac{1}{9}-\frac{1}{19}\right)\left[1.8052+.8333-\frac{2}{3(12.2143)}\right]$$

$$w=.9889$$

and thus $F\approx e^{2w}=7.23$. The correct answer is $F=7.180$.

Approximating the Non-Central F-Distribution

Example 25. Compute $P(3.71|3,10,4)$ using the approximation **26.6.27** to the non-central F-distribution.

Using **26.6.27** with $\nu_1=3$, $\nu_2=10$, $\lambda=4$, $F'=3.71$ we have

$$x=\frac{\left[\left(\frac{3}{3+4}\right)(3.71)\right]^{1/3}\left[1-\frac{2}{9(10)}\right]-\left[1-\frac{2}{9}\frac{(3+8)}{(3+4)^2}\right]}{\left[\frac{2}{9}\frac{3+8}{(3+4)^2}+\frac{2}{9(10)}\left[\left(\frac{3}{3+4}\right)(3.71)\right]^{2/3}\right]^{\frac{1}{2}}}=.675$$

and interpolating in **Table 26.1** gives

$$P(3.71|3,10,4)\approx P(.675)=.750$$

The exact answer is $P(3.71|3,10,4)=.745$.

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Tables

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