$$I_x(10,6)=Q(F|12,20)=.10 \text{ where } x=\frac{20}{20+12F}$$

From **Table 26.9** the upper 10 percent point of F with 12 and 20 degrees of freedom is $F=1.89$. Hence

$$x=\frac{20}{20+12(1.89)}=.469$$

The correct value to 4D is $x=.4683$.

Calculation of $I_x(a, b)$ for a or b Small Integers

Example 17. Calculate $I_{.10}(3, 20)$.

Values of $I_x(a, b)$ for small integral a or b can conveniently be calculated using **26.5.6** or **26.5.7.** Using **26.5.6** we have

$$1-I_{.90}(20,3)=\frac{(.9)^{20}}{B(3,20)}\left\{\sum_{i=0}^{2}(-1)^i\binom{2}{i}\frac{.9^i}{20+i}\right\}$$

$$=\frac{.121576}{.216450\times10^{-3}}(.110390\times10^{-2})=.620040$$

Binomial Distribution

Example 18. Find the value of p which satisfies

$$\sum_{s=0}^{20}\binom{50}{s}p^sq^{50-s}=.95,\qquad q=1-p$$

using **26.5.24** and **Table 26.9.**

* Combining **26.5.24** and **26.5.28** we have

$$\sum_{s=a}^{n}\binom{n}{s}p^sq^{n-s}=Q(F|\nu_1, \nu_2)$$

where

$$\nu_1=2(n-a+1), \nu_2=2(a), \text{ and } p=\frac{a}{a+(n-a+1)F}$$

Hence

$$\sum_{s=0}^{20}\binom{50}{s}p^sq^{50-s}=1-\sum_{s=21}^{50}\binom{50}{s}p^sq^{50-s}$$
$$=1-Q(F|60,42)=.95$$

Harmonic interpolation on ν_2 in the table for which $Q(F|\nu_1,\nu_2)=.05$ results in $F=1.624$ for $\nu_1=60$, $\nu_2=42$, and thus $p=\frac{42}{42+60(1.624)}=.301$. The correct answer to 4D is $p=.3003$.

Approximating the Incomplete Beta Function

Example 19. Find $I_{.60}(16, 10.5)$ using **26.5.21.**

Values of $I_x(a, b)$ can conveniently be calculated with good accuracy using the approximation given by **26.5.20** or **26.5.21.** For this example $(a+b-1)(1-x)=10.20$ which is greater than .8 and hence **26.5.21** will be used. Thus

$$w_1=[(10.5)(.60)]^{1/3}=1.8469, w_2=[16(.4)]^{1/3}=1.8566$$

$$y=\frac{3[(1.8469)(.98942)-(1.8566)(.99306)]}{\left[\frac{(1.8469)^2}{10.5}+\frac{(1.8566)^2}{16}\right]^{\frac{1}{2}}}=-.0668$$

and interpolating in **Table 26.1** gives

$$P(-.0668)=1-P(.0668)=.47336$$

The answer correct to 5D is $I_{.60}(16, 10.5)=.47332$.

Interpolation for F in Table 26.9

Example 20. Find the value of F for which

$Q(F|7, 20)=.05$ using **Table 26.9.**

Interpolation in **Table 26.9** is approximately linear when the reciprocals of the degrees of freedom (ν_1, ν_2) are used as the interpolating variable. For this example it is only necessary to interpolate with respect to $1/\nu_1$. Thus linear interpolation on $1/\nu_1$ results in $F=2.51$ which is the correct interpolate.

Calculation of F for $Q(F|\nu_1,\nu_2)>.50$

Example 21. Find the value of F for which $Q(F|4, 8)=.90$ using **26.6.9** and **Table 26.9.**

Table 26.9 only tabulates values of F for which $Q(F|\nu_1, \nu_2)=p$ where $p=.500$, .250, .100, .050, .025, .010, .005, .001. However making, use of **Table 26.9** we can find the values of F_p for which $p=.75$, .9, .95, .975, .99, .995, .999. For this example we have

$$F_{.90}(4,8)=\frac{1}{F_{.10}(8,4)}$$

and referring to the table for which $Q(F|\nu_1, \nu_2)=.10$ gives $F_{.10}(8, 4)=3.95$ and thus $F_{.90}(4, 8)=\frac{1}{3.95}=.253$.

Calculation of $Q(F|\nu_1,\nu_2)$ for Small Integral ν_1 or ν_2

Example 22. Compute $Q(2.5|4, 15)$ using **26.6.4.**

Values of $Q(F|\nu_1, \nu_2)$ can be readily computed for small ν_1 or ν_2 using the expansions **26.6.4** to **26.6.8** inclusive. We have using **26.6.4**

$$x=\frac{15}{15+4(2.50)}=.60$$

and

$$Q(2.50|4,15)=(.6)^{7.5}\left[1+\frac{15}{2}(.4)\right]=.086\,735$$

*See page II.