$$h_{72}=\frac{60}{72}(-.0006)=-.00049$$

Thus

$$Q(84|72)=Q(1.0046-.0005)=Q(1.0041)$$
$$=1-P(1.0041)=.15766$$

4. Square root approximation 26.4.13

Using the square root approximation we have $Q(84|72)=Q(x)$ where

$$x=\sqrt{2(84)}-\sqrt{2(72)-1}=1.0032.$$

This results in

$$Q(84|72)=Q(1.0032)=1-P(1.0032)=.15788$$

The value correct to 6D is $Q(84|72)=.157653$. Generally the improved cube root approximation will be correct with a maximum error of a few units in the fifth decimal and is recommended for calculations which are outside the range of **Table 26.7.**

Calculation of χ^2 for $Q(\chi^2|\nu)$ Outside the Range of Table 26.8

Example 13. Find the value of χ^2 for which $Q(\chi^2|144)=.01$.

Since $\nu=144$ is outside the range of **Table 26.8,** we can compute it by using (1) the Cornish-Fisher asymptotic expansion **26.2.50,** for χ^2, (2) the cube approximation **26.4.17,** (3) the improved cube approximation **26.4.18,** or (4) the square approximation **26.4.16.** We shall compute the value by all four methods.

1. Cornish-Fisher asymptotic expansion 26.2.50

The Cornish-Fisher asymptotic expansion for χ^2 with $\nu=144$ can be written as

$$\chi^2\sim 144+12\sqrt{2}x+4h_1(x)+\frac{4\sqrt{2}}{12}[3h_2(x)+2h_{11}(x)]$$
$$+\frac{8}{12^2}[6h_3(x)+3h_{12}(x)+2h_{111}(x)]+\frac{16\sqrt{2}}{12^3}[30h_4(x)$$
$$+9h_{22}(x)+12h_{13}(x)+6h_{112}(x)+4h_{1111}(x)]$$

Hence using the auxiliary table following **26.2.51** with $p=.01$ we have

144.0000
39.4794
2.9413
−.0242
−.0019
+.0002
$\chi^2=186.395$

2. Cube approximation 26.4.17

Taking $\chi_{.01}=2.32635$ we have

$$\chi^2=144\left\{\left[1-\frac{2}{9(144)}\right]+(2.32635)\sqrt{\frac{2}{9(144)}}\right\}^3=186.405$$

3. Improved cube approximation 26.4.18

From the table for h_{60} we obtain using linear interpolation with $x=2.33$ (approximately)

$$h_{60}=.0012 \text{ and thus } h_{144}=\frac{60}{144}(.0012)=.00049$$

Hence

$$\chi^2=144\left[1-\frac{2}{9(144)}+(2.32635-.00049)\sqrt{\frac{2}{9(144)}}\right]^3=186.394$$

4. Square approximation 26.4.16

$$\chi^2=\frac{1}{2}[2.32635+\sqrt{2(144)-1}]^2=185.616$$

The correct answer to 3D is $\chi^2=186.394$. Generally the improved cube approximation will give results correct in the second or third decimal for $\nu>30$.

Calculation of the Incomplete Gamma Function

Example 14. Find the value of

$$\gamma(2.5,.9)=\int_0^{.9}t^{1.5}e^{-t}dt$$

making use of **26.4.19** and **Table 26.7.**

Using **26.4.19** we have

$$\gamma(2.5,.9)=\Gamma(2.5)P(1.8|5)=\Gamma(2.5)[1-Q(1.8|5)]$$

$$\gamma(2.5,.9)=\frac{3}{4}\sqrt{\pi}[1-.87607]=.16475$$

Poisson Distribution

Example 15. Find the value of m for which

$$\sum_{i=0}^{3}e^{-m}\frac{m^i}{i!}=.99$$

using **26.4.21** and **Table 26.8.**

From **Table 26.8** with $\nu=2c=8$ and $Q=.99$ we have $\chi^2=1.646482$. Hence $m=\chi^2/2=.823241$.

Inverse of the Incomplete Beta Function

Example 16. Find the value of x for which $I_x(10, 6)=.10$ using **Table 26.9** and **26.5.28.** * Using **26.5.28** we have

*See page II.