Since $R<1$, the approximation **26.3.25** is used. This results in

$$P(R^2|2, r^2)=\frac{2(.54)^2}{4+(.54)^2}\exp\frac{-2(1.21)^2}{4+(.54)^2}$$
$$=(.1359)e^{-.6823}=.06869$$

The answer to 5D is .06870.

Interpolation for $Q(\chi^2|\nu)$

Example 11. Find $Q(25.298|20)$ using the interpolation formula given with **Table 26.7.**

Taking $\chi^2=25$, $\theta=.298$ and applying the interpolation formula results in

$$Q(25.298|20)=\frac{1}{8}\{Q(25|16)\theta^2+Q(25|18)(4\theta-2\theta^2)+Q(25|20)(8-4\theta+\theta^2)\}$$
$$=\frac{1}{8}\{(.06982)(.088804)+(.12492)(1.014392)+(.20143)(6.896804)\}$$
$$=.19027$$

A less accurate interpolate may be obtained by setting θ^2 equal to zero in the above formula. This results in the value .19003. The correct value to 6D is $Q(25.298|20)=.190259$.

On the other hand if $\chi^2=25.298$ is assumed to have an error of $\pm 5\times 10^{-4}$, then how large an error arises in $Q(\chi^2|\nu)$? Denoting the error in χ^2 by $\Delta\chi^2$ and the resulting error in $Q(\chi^2|\nu)$ by $\Delta Q(\chi^2|\nu)$, we then have the approximate relationship

$$\Delta Q(\chi^2|\nu)\approx\frac{\partial Q(\chi^2|\nu)}{\partial\chi^2}\,\Delta\chi^2$$

Using **26.4.8** we can write

$$\frac{\partial Q(\chi^2|\nu)}{\partial\chi^2}=\frac{1}{2}\,[Q(\chi^2|\nu-2)-Q(\chi^2|\nu)]$$

and

$$\Delta Q(\chi^2|\nu)\approx\frac{1}{2}\,[Q(\chi^2|\nu-2)-Q(\chi^2|\nu)]\Delta\chi^2$$

For practical purposes it is sufficient to evaluate the derivative to one or two significant figures. Consequently we can write

$$\frac{\partial Q(\chi^2|\nu)}{\partial\chi^2}\approx\frac{\partial Q(\chi_0^2|\nu)}{\partial\chi^2}$$

where χ_0^2 is the closest value to χ^2 for which Q is tabulated. Hence

$$\Delta Q(\chi^2|\nu)\approx\frac{1}{2}\,[Q(\chi_0^2|\nu-2)-Q(\chi_0^2|\nu)]\Delta\chi^2$$

For this example $\Delta\chi^2=\pm 5\times 10^{-4}$ and $\chi_0^2=25$. This results in

$$\Delta Q(\chi^2|\nu)=\frac{1}{2}\,(-.076)(\pm 5)10^{-4}=\pm 2\times 10^{-5}$$

as the possible error in $Q(\chi^2|\nu)$.

Calculation of $Q(\chi^2|\nu)$ Outside the Range of Table 26.7

Example 12. Find the value of $Q(84|72)$.

Since this value is outside the range of **Table 26.7** we can approximate $Q(84|72)$ by (1) using the Edgeworth expansion for $Q(\chi^2|\nu)$ given in **Example 6,** (2) the cube root approximation **26.4.14,** (3) the improved cube root approximation **26.4.15** or (4) the square root approximation **26.4.13.** The results of using all four methods are presented below:

1. Edgeworth expansion

The successive terms of the Edgeworth expansion for the distribution of chi-square result in

$$\begin{aligned}1-Q(84|72)=&.841345\\&.000000\\&.001120\\ \hline &.842465\end{aligned}$$

Hence $Q(84|72)=.15754$.

The successive terms of the Edgeworth expansion for the distribution of $\sqrt{2\chi^2}$ result in

$$\begin{aligned}1-Q(84|72)=&.842544\\&-.000034\\&-.000138\\ \hline &.842372\end{aligned}$$

Hence $Q(84|72)=.15764$.

2. Cube root approximation 26.4.14

Using the cube root approximation we have

$$Q(84|72)=Q(x)$$

where

$$x=\frac{\left(\frac{84}{72}\right)^{1/3}\left[1-\frac{2}{9(72)}\right]}{\left[\frac{2}{9(72)}\right]^{\frac{1}{2}}}=1.0046$$

This results in $Q(84|72)=Q(1.0046)=1-P(1.0046)=.15754$.

3. Improved cube root approximation 26.4.15

The improved cube root approximation involves calculating a correction factor h_ν to x. Linearly interpolating for h_{60} (which appears below **26.4.15**) with $x=1.0046$ results in $h_{60}=-.0006$ and hence