$$t=-.00003\ 60167\ 31129$$

and carrying fifteen decimals we have the successive terms

+	1.96000	00000	00000
−	.00003	60167	31129
+		12	71261
−			68
			0
+	1.95996	39845	40064

Edgeworth Asymptotic Expansion

Example 6. Find the Edgeworth asymptotic expansion **26.2.49** for the c.d.f. of chi-square.

Method 1. Expansion for χ^2

Let

$$Q(\chi^2|\nu)=1-F(t)$$

where

$$t=\frac{\chi^2-\nu}{(2\nu)^{\frac{1}{2}}}$$

Since the values of γ_1 and γ_2 **26.4.33** are

$$\gamma_1=2\sqrt{2}/\nu^{\frac{1}{2}}$$

$$\gamma_2=12/\nu,$$

we obtain, by using the first two bracketed terms of **26.2.49**

$$F(t)\sim P(t)-\frac{1}{\nu^{\frac{1}{2}}}\left[\frac{\sqrt{2}}{3}Z^{(2)}(t)\right]+\frac{1}{\nu}\left[\frac{1}{2}Z^{(3)}(t)+\frac{1}{9}Z^{(5)}(t)\right]$$

The Edgeworth expansion is an asymptotic expansion in terms of derivatives of the normal distribution function. It is often possible to transform a random variable so that the distribution of the transformed random variable more closely approximates the normal distribution function than does the distribution of the original random variable. Hence for the same number of terms, greater accuracy may be achieved by using the transformed variable in the expansion. Since the distribution of $\sqrt{2\chi^2}$ is more closely approximated by a normal distribution than χ^2 itself (as judged by a comparison of the values of γ_1 and γ_2), we would expect that the Edgeworth asymptotic expansion of $\sqrt{2\chi^2}$ would be superior to that of χ^2.

Method 2. Expansion for $\sqrt{2\chi^2}$. Let

$$Q(\chi^2|\nu)=1-F(t)=1-F\left(\frac{\sqrt{2\chi^2}-(2\nu-1)^{\frac{1}{2}}}{\left(1-\frac{1}{4\nu}\right)^{\frac{1}{2}}}\right)$$

where $(2\nu-1)^{\frac{1}{2}}$ and $1-\frac{1}{4\nu}$ are the mean and variance to terms of order ν^{-2} of $\sqrt{2\chi^2}$ (see **26.4.34**). The values of γ_1 and γ_2 for $\sqrt{2\chi^2}$ are

$$\gamma_1\approx\frac{1}{\sqrt{2\nu}}\left[1+\frac{5}{8\nu}\right]\qquad\gamma_2\approx\frac{3}{4\nu^2}$$

Thus we obtain

$$F(t)\sim P(t)-\frac{1}{\nu^{\frac{1}{2}}}\left[\frac{\sqrt{2}}{12}\left(1+\frac{5}{8\nu}\right)Z^{(2)}(t)\right]+\frac{1}{\nu}\left[\frac{1}{32\nu}Z^{(3)}(t)+\frac{1}{144}\left(1+\frac{5}{8\nu}\right)^2Z^{(5)}(t)\right]$$

For numerical examples using these expansions see **Example 12.**

Calculation of $L(h, k, \rho)$

Example 7. Find $L(.5, .4, .8)$. Using **26.3.20**

$$\sqrt{h^2-2\rho hk+k^2}=\sqrt{.09}=.3$$

$$L(.5,.4,.8)=L(.5,0,0)+L(.4,0,-.6)$$

Reference to **Figure 26.2** yields

$$L(.5,0,0)+L(.4,0,-.6)=.16+.08=.24$$

The answer to 3D is $L(.5, .4, .8)=.250$.

Calculation of the Bivariate Normal Probability Function

Example 8. Let X and Y follow a bivariate normal distribution with parameters $m_x=3$, $m_y=2$, $\sigma_x=4$, $\sigma_y=2$, and $\rho=-.125$. Find the value of $P_r\{X\geq 2,\ Y\geq 4\}$ using **26.3.20** and **Figures 26.2, 26.3.**

Since $P_r\{X\geq h, Y\geq k\}=L\left(\frac{h-m_x}{\sigma_x},\frac{k-m_y}{\sigma_y},\rho\right)$ we have $P\{X\geq 2,\ Y\geq 4\}=L(-.25, 1, -.125)$. Using **26.3.20**

$$L(-.25,1,-.125)=L(-.25,0,.969)+L(1,0,.125)-\frac{1}{2}$$

Figure 26.2 only gives values for $h>0$, however, using the relationship **26.3.8** with $k=0$, $L(-h,0,\rho)=\frac{1}{2}-L(h,0,-\rho)$ and thus $L(-.25,0,.969)=\frac{1}{2}-L(.25,0,-.969)$. Therefore $L(-.25,1,-.125)=-L(.25,0,-.969)+L(1,0,.125)=-.01+.09=.08$.

The answer to 3D is $L(-.25, 1., -.125)=.080$.