Properties of Chi-Square, Non-Central Chi-Square, and Related Quantities

$$a=\nu+\lambda \qquad b=\frac{\lambda}{\nu+\lambda}$$

$$\psi(z)=\frac{d}{dz}\ln\Gamma(z), \qquad \psi^{\nu}(z)=\frac{d^{\nu}}{dz^{\nu}}\psi(z)$$

	Variable	*Mean*	*Variance*	*Coefficient of skewness* (γ_1)	*Coefficient of excess* (γ_2)
26.4.33	χ^2	ν	2ν	$\frac{2^{3/2}}{\sqrt{\nu}}$	$12\nu^{-1}$
26.4.34	$\sqrt{2\chi^2}$	$(2\nu-1)^{\frac{1}{2}}\{1+[16\nu(\nu-1)]^{-1}\}+O(\nu^{-7/2})$	$1-\frac{1}{4\nu}-\frac{1}{8\nu^2}+\frac{5}{64\nu^3}-O(\nu^{-4})$	$\frac{1}{\sqrt{2\nu}}\left[1+\frac{5}{8\nu}-\frac{1}{128\nu^2}\right]+O(\nu^{-7/2})$	$\frac{3}{2^3}\frac{1}{\nu^2}\left[1+\frac{3}{2\nu}\right]+O(\nu^{-4})$
26.4.35	$(\chi^2/\nu)^{1/3}$	$1-\frac{2}{3^2\nu}+\frac{80}{3^7\nu^2}+O(\nu^{-4})$	$\frac{2}{3^2\nu}-\frac{104}{3^7\nu^2}+O(\nu^{-4})$	$\frac{2^{7/2}}{3^3\nu^{3/2}}\left[1+\frac{8}{3^2\nu}\right]+O(\nu^{-7/2})$	$-\frac{4}{9\nu}\left[1+\frac{16}{9\nu}\right]+O(\nu^{-3})$
26.4.36	$\ln(\chi^2/\nu)$	$\psi\left(\frac{\nu}{2}\right)-\ln\left(\frac{\nu}{2}\right)=-\frac{1}{\nu}-\frac{1}{3\nu^2}+O(\nu^{-4})$	$\psi'\left(\frac{\nu}{2}\right)=\frac{2}{\nu-1}\left[1-\frac{1}{3(\nu-1)^2}\right]+O((\nu-1)^{-4})$	$\frac{\psi''\left(\frac{\nu}{2}\right)}{\psi'\left(\frac{\nu}{2}\right)^{3/2}}=-\sqrt{\frac{2}{\nu-1}}\left[1-\frac{1}{2(\nu-1)^2}\right]+O((\nu-1)^{-4.5})$	$\frac{\psi^{(3)}\left(\frac{\nu}{2}\right)}{\psi'\left(\frac{\nu}{2}\right)^2}=\frac{4}{\nu-1}\left[1+\frac{4}{3(\nu-1)^2}\right]+O((\nu-1)^{-4})$
26.4.37	χ'^2	a	$2a(1+b)$	$\left(\frac{2}{1+b}\right)^{3/2}(1+2b)a^{-\frac{1}{2}}$	$\frac{12}{a}\frac{(1+3b)}{(1+b)^2}$
26.4.38	$\sqrt{2\chi'^2}$	$[2a-(1+b)]^{\frac{1}{2}}+O(a^{-3/2})$	$(1+b)-\frac{a^{-1}}{4}[3b+(1+b)(1-7b)]+O(a^{-2})$	$\frac{a^{-\frac{1}{2}}(1-b)(1+3b)}{2^{\frac{1}{2}}(1+b)^{3/2}}+O(a^{-1})$	$\frac{3b(b+2)}{(1+b)^2}+O(a^{-2})$
26.4.39	$(\chi'^2/a)^{1/3}$	$1-\frac{2}{3^2}\frac{1+b}{a}-\frac{40}{3^4}\frac{b^2}{a^2}+O(a^{-3})$	$\frac{2}{9}a^{-1}(1+b)+\frac{16}{27}a^{-2}b^2+O(a^{-3})$	$\left(\frac{2}{1+b}\right)^{3/2}b^2a^{-\frac{1}{2}}+O(a^{-3/2})$	$-\frac{4}{3^2}\frac{(1+3b+12b^2-44b^3)}{a(1+b)^2}-O(a^{-2})$