26.4.22 Pearson Type III

$$\left[\frac{ab}{e}\right]^{ab}\int_{-a}^{x}\left(1+\frac{t}{a}\right)^{ab}e^{-bt}dt=P(\chi^2|\nu)$$

$$\nu=2ab+2,\ \chi^2=2b(x+a)$$

26.4.23 Incomplete moments of Normal distribution

$$\int_0^x t^n Z(t)dt=\begin{cases}(n-1)!!\,\dfrac{P(\chi^2|\nu)}{2} & (n \text{ even})\\[2ex] \dfrac{(n-1)!!}{\sqrt{2\pi}}P(\chi^2|\nu) & (n \text{ odd})\end{cases}$$

$$\chi^2=x^2,\ \nu=n+1$$

26.4.24 Generalized Laguerre Polynomials

$$n!L_n^{(\alpha)}(x)=\frac{\sum_{j=0}^{n+1}(-1)^{n+j}\binom{n+1}{j}Q(\chi^2|\nu+2-2j)}{2^n[Q(\chi^2|\nu+2)-Q(\chi^2|\nu)]}$$

$$x=\chi^2/2,\ \alpha=\nu/2$$

Non-Central χ^2 Distribution Function

26.4.25

$$P(\chi'^2|\nu,\lambda)=\sum_{j=0}^{\infty}e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}P(\chi'^2|\nu+2j)$$

where $\lambda\geq 0$ is termed the non-centrality parameter.

Relation of Non-Central χ^2 Distribution With $\nu=2$ to the Integral of Circular Normal Distribution ($\sigma^2=1$) Over an Offset Circle Having Radius R and Center a Distance $r=\sqrt{\lambda}$ From the Origin. (See 26.3.24–26.3.27.)

26.4.26

$$\iint_A g(x,y,0)\,dxdy=P(\chi^2=R^2|\nu=2,\lambda)$$

$$=1-\sum_{j=0}^{\infty}\frac{e^{-\lambda/2}\lambda^j}{2^j j!}Q(R^2|2+2j)$$

Approximations to the Non-Central χ^2 Distribution

$$a=\nu+\lambda \qquad b=\frac{\lambda}{\nu+\lambda}$$

	Approximating Function	*Approximation*
26.4.27	χ^2 distribution	$P(\chi'^2\|\nu,\lambda)\approx P\left(\frac{\chi^2}{1+b}\middle\|\nu^*\right),\quad \nu^*=\frac{a}{1+b}$
26.4.28	Normal distribution	$P(\chi'^2\|\nu,\lambda)\approx P(x),\quad x=\frac{(\chi'^2/a)^{1/3}-\left[1-\frac{2}{9}\left(\frac{1+b}{a}\right)\right]}{\sqrt{\frac{2}{9}\left(\frac{1+b}{a}\right)}}$
26.4.29	Normal distribution	$P(\chi'^2\|\nu,\lambda)\approx P(x),\quad x=\left[\frac{2\chi'^2}{1+b}\right]^{\frac{1}{2}}-\left[\frac{2a}{1+b}-1\right]^{\frac{1}{2}}$

Approximations to the Inverse Function of Non-Central χ^2 Distribution

If $Q(\chi_p'^2|\nu,\lambda)=p$, $Q(\chi_p^2|\nu^*)=p$, and $Q(x_p)=p$ then

	Approximating Variable	*Approximation to the Inverse Function*
26.4.30	χ^2	$\chi_p'^2\approx(1+b)\chi_p^2$
26.4.31	Normal	$\chi_p'^2\approx\frac{1+b}{2}\left[x_p+\sqrt{\frac{2a}{1+b}-1}\right]^2$
26.4.32	Normal	$\chi_p'^2\approx a\left[x_p\sqrt{\frac{2}{9}\left(\frac{1+b}{a}\right)}+1-\frac{2}{9}\left(\frac{1+b}{a}\right)\right]^3$