Series Expansions

26.4.4

$$Q(\chi^2|\nu)=2Q(\chi)+2Z(\chi)\sum_{r=1}^{\frac{\nu-1}{2}}\frac{\chi^{2r-1}}{1\cdot3\cdot5\ldots(2r-1)}$$

(ν odd) and $\chi=\sqrt{\chi^2}$

26.4.5

$$Q(\chi^2|\nu)=\sqrt{2\pi}Z(\chi)\left\{1+\sum_{r=1}^{\frac{\nu-2}{2}}\frac{\chi^{2r}}{2\cdot4\ldots(2r)}\right\}$$

(ν even)

26.4.6

$$P(\chi^2|\nu)=\left(\frac{1}{2}\chi^2\right)^{\nu/2}\frac{e^{-\chi^2/2}}{\Gamma\left(\frac{\nu+2}{2}\right)}$$

$$*\quad\left\{1+\sum_{r=1}^{\infty}\frac{\chi^{2r}}{(\nu+2)(\nu+4)\cdots(\nu+2r)}\right\}$$

26.4.7 $$P(\chi^2|\nu)=\frac{1}{\Gamma\left(\frac{\nu}{2}\right)}\sum_{n=0}^{\infty}\frac{(-1)^n(\chi^2/2)^{\frac{\nu}{2}+n}}{n!\left(\frac{\nu}{2}+n\right)}$$

Recurrence and Differential Relations

26.4.8 $$Q(\chi^2|\nu+2)=Q(\chi^2|\nu)+\frac{(\chi^2/2)^{\nu/2}e^{-\chi^2/2}}{\Gamma\left(\frac{\nu}{2}+1\right)}$$

26.4.9 $$\frac{\partial^m Q(\chi^2|\nu)}{\partial(\chi^2)^m}=\frac{1}{2^m}\sum_{j=0}^{m}\binom{m}{j}(-1)^{m+j}Q(\chi^2|\nu-2j)$$

Continued Fraction

26.4.10 $$*Q(\chi^2|\nu)=\frac{(\chi^2)^{\nu/2}e^{-\chi^2/2}}{2^{\nu/2}\Gamma(\nu/2)}$$

$$\left\{\frac{1}{\chi^2/2+}\;\frac{1-\nu/2}{1+}\;\frac{1}{\chi^2/2+}\;\frac{2-\nu/2}{1+}\;\frac{2}{\chi^2/2+}\ldots\right\}$$

Asymptotic Distribution for Large ν

26.4.11 $P(\chi^2|\nu)\sim P(x)$ where $x=\dfrac{\chi^2-\nu}{\sqrt{2\nu}}$

Asymptotic Expansions for Large χ^2

26.4.12

$$Q(\chi^2|\nu)\sim\frac{(\chi^2)^{\frac{\nu}{2}-1}e^{-\chi^2/2}}{2^{\nu/2}\Gamma(\nu/2)}\sum_{j=0}^{\infty}(-1)^j\frac{\Gamma\left(1-\frac{\nu}{2}+j\right)}{\Gamma\left(1-\frac{\nu}{2}\right)}\frac{2^{j+1}}{(\chi^2)^j}$$

*See page II.

Approximations to the Chi-Square Distribution for Large ν

26.4.13

Approximation		Condition
$Q(\chi^2|\nu)\approx Q(x_1)$,	$x_1=\sqrt{2\chi^2}-\sqrt{2\nu-1}$	$(\nu>100)$

26.4.14

$$Q(\chi^2|\nu)\approx Q(x_2),\qquad x_2=\frac{(\chi^2/\nu)^{1/3}-\left(1-\frac{2}{9\nu}\right)}{\sqrt{2/9\nu}}\qquad(\nu>30)$$

26.4.15

$$Q(\chi^2|\nu)\approx Q(x_2+h_\nu),\qquad h_\nu=\frac{60}{\nu}h_{60}\qquad(\nu>30)$$

Values of h_{60}

x	h_{60}	x	h_{60}	x	h_{60}
−3.5	−.0118	−1.0	+.0006	+1.5	−.0005
−3.0	−.0067	−.5	.0006	2.0	+.0002
−2.5	−.0033	.0	+.0002	2.5	.0017
−2.0	−.0010	+.5	−.0003	3.0	.0043
−1.5	+.0001	1.0	−.0006	3.5	.0082

Approximations for the Inverse Function for Large ν

If $Q(\chi_p^2|\nu)=p$ and $Q(x_p)=1-P(x_p)=p$, then

	Approximation	Condition
26.4.16	$\chi_p^2\approx\frac{1}{2}\left\{x_p+\sqrt{2\nu-1}\right\}^2$	$(\nu>100)$
26.4.17	$\chi_p^2\approx\nu\left\{1-\frac{2}{9\nu}+x_p\sqrt{\frac{2}{9\nu}}\right\}^3$	$(\nu>30)$
26.4.18	$\chi_p^2\approx\nu\cdot\left\{1-\frac{2}{9\nu}+(x_p-h_\nu)\sqrt{\frac{2}{9\nu}}\right\}^3$	$(\nu>30)$

where h_ν is given by **26.4.15.**

Relation to Other Functions

26.4.19 Incomplete gamma function

$$\frac{\gamma(a,x)}{\Gamma(a)}=P(\chi^2|\nu),\qquad\nu=2a,\ \chi^2=2x$$

$$\frac{\Gamma(a,x)}{\Gamma(a)}=Q(\chi^2|\nu)$$

26.4.20 Pearson's incomplete gamma function

$$I(u,p)=\frac{1}{\Gamma(p+1)}\int_0^{u\sqrt{p+1}}t^pe^{-t}dt=P(\chi^2|\nu)$$

$$\nu=2(p+1),\ \chi^2=2u\sqrt{p+1}$$

26.4.21 Poisson distribution

$$Q(\chi^2|\nu)=\sum_{j=0}^{c-1}e^{-m}\frac{m^j}{j!},\qquad c=\frac{\nu}{2},\ m=\frac{\chi^2}{2},\ (\nu\text{ even})$$

$$Q(\chi^2|\nu)-Q(\chi^2|\nu-2)=e^{-m}\frac{m^{c-1}}{(c-1)!}$$