26.2.21

$$Z(x)=(b_0+b_2x^2+b_4x^4+b_6x^6+b_8x^8+b_{10}x^{10})^{-1}+\epsilon(x)$$

$$|\epsilon(x)|<2.3\times10^{-4}$$

$b_0=2.50523\ 67$ $\quad b_6=\ .13064\ 69$

$b_2=1.28312\ 04$ $\quad b_8=-.02024\ 90$

$b_4=\ .22647\ 18$ $\quad b_{10}=\ .00391\ 32$

Rational Approximations [7] for x_p where $Q(x_p)=p$

$$0<p\leq .5$$

26.2.22

$$x_p=t-\frac{a_0+a_1t}{1+b_1t+b_2t^2}+\epsilon(p),\qquad t=\sqrt{\ln\frac{1}{p^2}}$$

$$|\epsilon(p)|<3\times10^{-3}$$

$a_0=2.30753$ $\quad b_1=.99229$

$a_1=\ .27061$ $\quad b_2=.04481$

26.2.23

$$x_p=t-\frac{c_0+c_1t+c_2t^2}{1+d_1t+d_2t^2+d_3t^3}+\epsilon(p),\qquad t=\sqrt{\ln\frac{1}{p^2}}$$

$$|\epsilon(p)|<4.5\times10^{-4}$$

$c_0=2.515517$ $\quad d_1=1.432788$

$c_1=\ .802853$ $\quad d_2=\ .189269$

$c_2=\ .010328$ $\quad d_3=\ .001308$

Bounds Useful as Approximations to the Normal Distribution Function

26.2.24

$$P(x)\leq\begin{cases}P_1(x)=\frac{1}{2}+\frac{1}{2}\,(1-e^{-2x^2/\pi})^{\frac{1}{2}} & (x>0)\\[2ex] P_2(x)=1-\frac{(4+x^2)^{\frac{1}{2}}-x}{2}\,(2\pi)^{-\frac{1}{2}}e^{-x^2/2} & (x>1.4)\end{cases}$$

26.2.25

$$P(x)\geq\begin{cases}P_3(x)=\frac{1}{2}+\frac{1}{2}\left(1-e^{-2x^2/\pi}-\frac{2(\pi-3)}{3\pi^2}\,x^4e^{-x^2/2}\right)^{\frac{1}{2}} & (x>0)\\[2ex] P_4(x)=1-\frac{1}{x}\,(2\pi)^{-\frac{1}{2}}e^{-x^2/2} & (x>2.2)\end{cases}$$

See **Figure 26.1** for error curves.

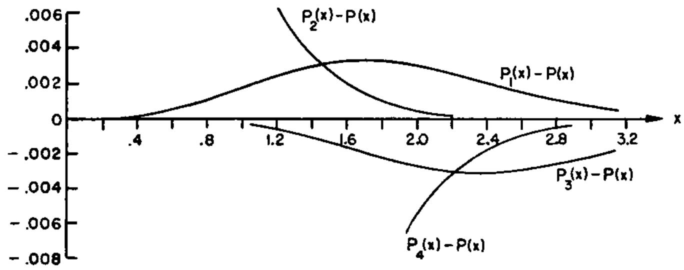

FIGURE 26.1. *Error curves for bounds on normal distribution.*

Derivatives of the Normal Probability Density Function

26.2.26

$$Z^{(m)}(x)=\frac{d^m}{dx^m}\,Z(x)$$

Differential Equation

26.2.27 $\quad Z^{(m+2)}(x)+xZ^{(m+1)}(x)+(m+1)Z^{(m)}(x)=0$

Value at $x=0$

26.2.28

$$Z^{(m)}(0)=\begin{cases}\dfrac{(-1)^{m/2}m!}{\sqrt{2\pi}2^{m/2}\left(\frac{m}{2}\right)!} & \text{for } m=2r, r=0, 1, \ldots\\[2ex] 0 & \text{for odd } m>0\end{cases}$$