Power Series ($x \geq 0$)

26.2.10 $$P(x)=\frac{1}{2}+\frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{n!2^n(2n+1)}$$

26.2.11
$$P(x)=\frac{1}{2}+Z(x)\sum_{n=0}^{\infty}\frac{x^{2n+1}}{1\cdot 3\cdot 5\ldots(2n+1)}$$

Asymptotic Expansions ($x>0$)

26.2.12

$$Q(x)=\frac{Z(x)}{x}\left\{1-\frac{1}{x^2}+\frac{1\cdot 3}{x^4}+\ldots+\frac{(-1)^n 1\cdot 3\ldots(2n-1)}{x^{2n}}\right\}+R_n$$

where

$$R_n=(-1)^{n+1}1\cdot 3\ldots(2n+1)\int_x^{\infty}\frac{Z(t)}{t^{2n+2}}\,dt$$

which is less in absolute value than the first neglected term.

26.2.13

$$Q(x)\sim\frac{Z(x)}{x}\left\{1-\frac{a_1}{x^2+2}+\frac{a_2}{(x^2+2)(x^2+4)}-\frac{a_3}{(x^2+2)(x^2+4)(x^2+6)}+\ldots\right\}$$

where $a_1=1$, $a_2=1$, $a_3=5$, $a_4=9$, $a_5=129$ and the general term is

$$a_n=c_0 1\cdot 3\ldots(2n-1)+2c_1 1\cdot 3\ldots(2n-3)+2^2c_2 1\cdot 3\ldots(2n-5)+\ldots+2^{n-1}c_{n-1}$$

and c_s is the coefficient of t^{n-s} in the expansion of $t(t-1)\ldots(t-n+1)$.

Continued Fraction Expansions

26.2.14

$$Q(x)=Z(x)\left\{\frac{1}{x+}\,\frac{1}{x+}\,\frac{2}{x+}\,\frac{3}{x+}\,\frac{4}{x+}\cdots\right\}\qquad(x>0)$$

26.2.15

$$Q(x)=\frac{1}{2}-Z(x)\left\{\frac{x}{1-}\,\frac{x^2}{3+}\,\frac{2x^2}{5-}\,\frac{3x^2}{7+}\,\frac{4x^2}{9-}\cdots\right\}\qquad(x\geq 0)$$

Polynomial and Rational Approximations[7] for $P(x)$ and $Z(x)$

$$0\leq x<\infty$$

26.2.16

$$P(x)=1-Z(x)(a_1t+a_2t^2+a_3t^3)+\epsilon(x),\qquad t=\frac{1}{1+px}$$

$$|\epsilon(x)|<1\times 10^{-5}$$

$p=.33267$ $\quad a_1=.43618\ 36$

$a_2=-.12016\ 76$

$a_3=.93729\ 80$

26.2.17

$$P(x)=1-Z(x)(b_1t+b_2t^2+b_3t^3+b_4t^4+b_5t^5)+\epsilon(x),\qquad t=\frac{1}{1+px}$$

$$|\epsilon(x)|<7.5\times 10^{-8}$$

$p=.23164\ 19$

$b_1=\ .31938\ 1530$ $\quad b_4=-1.82125\ 5978$

$b_2=-.35656\ 3782$ $\quad b_5=\ 1.33027\ 4429$

$b_3=\ 1.78147\ 7937$

26.2.18

$$P(x)=1-\frac{1}{2}(1+c_1x+c_2x^2+c_3x^3+c_4x^4)^{-4}+\epsilon(x)$$

$$|\epsilon(x)|<2.5\times 10^{-4}$$

$c_1=.196854$ $\quad c_3=.000344$

$c_2=.115194$ $\quad c_4=.019527$

26.2.19

$$P(x)=1-\frac{1}{2}(1+d_1x+d_2x^2+d_3x^3+d_4x^4+d_5x^5+d_6x^6)^{-16}+\epsilon(x)$$

$$|\epsilon(x)|<1.5\times 10^{-7}$$

$d_1=.04986\ 73470$ $\quad d_4=.00003\ 80036$

$d_2=.02114\ 10061$ $\quad d_5=.00004\ 88906$

$d_3=.00327\ 76263$ $\quad d_6=.00000\ 53830$

26.2.20 $$Z(x)=(a_0+a_2x^2+a_4x^4+a_6x^6)^{-1}+\epsilon(x)$$

$$|\epsilon(x)|<2.7\times 10^{-3}$$

$a_0=2.490895$ $\quad a_4=-.024393$

$a_2=1.466003$ $\quad a_6=\ .178257$

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[7] Based on approximations in C. Hastings, Jr., Approximations for digital computers. Princeton Univ. Press, Princeton, N.J., 1955 (with permission).