The joint probability of the event $X_1 \leq x_1, X_2 \leq x_2, \ldots, X_n \leq x_n$ is $F(x_1, x_2, \ldots x_n)$. Analogous to the one-dimensional case, *discrete* distributions assign all probability to an enumerable set of vectors $(x_1, x_2, \ldots, x_n)$ and *continuous* distributions are characterized by absolute continuity of $F(x_1, x_2, \ldots, x_n)$.

Characteristics of distribution functions: Moments, characteristic functions, cumulants

		Continuous distributions	Discrete distributions
26.1.3	n^{th} moment about origin	$\mu_n' = \int_{-\infty}^{\infty} x^n f(x)dx$	$\mu_n' = \sum_s x_s^n p_s$
26.1.4	mean	$m = \mu_1' = \int_{-\infty}^{\infty} x f(x)dx$	$m = \mu_1' = \sum_s x_s p_s$
26.1.5	variance	$\sigma^2 = \mu_2' - m^2 = \int_{-\infty}^{\infty} (x-m)^2 f(x)dx$	$\sigma^2 = \mu_2' - m^2 = \sum_s (x_s - m)^2 p_s$
26.1.6	n^{th} central moment	$\mu_n = \int_{-\infty}^{\infty} (x-m)^n f(x)dx$	$\mu_n = \sum_s (x_s - m)^n p_s$
26.1.7	expected value operator for the function $g(x)$	$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)dx$	$E[g(X)] = \sum_s g(x_s) p_s$
26.1.8	characteristic function of X	$\phi(t) = E(e^{itX}) = \int_{-\infty}^{\infty} e^{itx} f(x)dx$	$\phi(t) = E(e^{itX}) = \sum_s e^{itx_s} p_s$
26.1.9	characteristic function of $g(X)$	$\phi_g(t) = E(e^{itg(X)}) = \int_{-\infty}^{\infty} e^{itg(x)} f(x)dx$	$\phi_g(t) = E(e^{itg(X)}) = \sum_s e^{itg(x_s)} p_s$
26.1.10	inversion formula	$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\phi(t)dt$	$p_n = \frac{b}{2\pi}\int_{-\pi/b}^{\pi/b} e^{-itx_n}\phi(t)dt$ (lattice distributions only)

Relation of the Characteristic Function to Moments About the Origin

26.1.11
$$\phi^{(n)}(0) = \left[\frac{d^n}{dt^n}\phi(t)\right]_{t=0} = i^n \mu_n'$$

Cumulant Function

26.1.12
$$\ln \phi(t) = \sum_{n=0}^{\infty} \kappa_n \frac{(it)^n}{n!}$$

κ_n is called the n^{th} cumulant.

26.1.13 $\kappa_1 = m, \kappa_2 = \sigma^2, \kappa_3 = \mu_3, \kappa_4 = \mu_4 - 3\mu_2^2$

Relation of Central Moments to Moments About the Origin

26.1.14
$$\mu_n = \sum_{j=0}^{n} \binom{n}{j} (-1)^{n-j} \mu_j' m^{n-j}$$

Coefficients of Skewness and Excess

26.1.15
$$\gamma_1 = \frac{\kappa_3}{\kappa_2^{3/2}} = \frac{\mu_3}{\sigma^3} \quad \text{(skewness)}$$

26.1.16
$$\gamma_2 = \frac{\kappa_4}{\kappa_2^2} = \frac{\mu_4}{\sigma^4} - 3 \quad \text{(excess)}$$

Occasionally coefficients of skewness and excess (or kurtosis) are given by

26.1.17
$$\beta_1 = \gamma_1^2 = \left(\frac{\mu_3}{\sigma^3}\right)^2 \quad \text{(skewness)}$$

26.1.18
$$\beta_2 = \gamma_2 + 3 = \frac{\mu_4}{\sigma^4}$$
(excess or kurtosis)