Table 25.5 ABSCISSAS FOR EQUAL WEIGHT CHEBYSHEV INTEGRATION

$$\int_{-1}^{+1} f(x)dx \approx \frac{2}{n}\sum_{i=1}^{n} f(x_i)$$

Abscissas $= \pm x_i$

n	$\pm x_i$	n	$\pm x_i$	n	$\pm x_i$
2	0.57735 02692	5	0.83249 74870	7	0.88386 17008
			0.37454 14096		0.52965 67753
			0.00000 00000		0.32391 18105
					0.00000 00000
3	0.70710 67812				
	0.00000 00000				
				9	0.91158 93077
					0.60101 86554
		6	0.86624 68181		0.52876 17831
4	0.79465 44723		0.42251 86538		0.16790 61842
	0.18759 24741		0.26663 54015		0.00000 00000

Compiled from H. E. Salzer, Tables for facilitating the use of Chebyshev's quadrature formula, J. Math. Phys. **26**, 191-194, 1947 (with permission).

Table 25.6 ABSCISSAS AND WEIGHT FACTORS FOR LOBATTO INTEGRATION

$$\int_{-1}^{+1} f(x)dx \approx w_1 f(-1) + \sum_{i=2}^{n-1} w_i f(x_i) + w_n f(1)$$

n	Abscissas $= \pm x_i$	Weight Factors $= w_i$	n	$\pm x_i$	w_i
			7	1.00000 000	0.04761 904
				0.83022 390	0.27682 604
				0.46884 879	0.43174 538
				0.00000 000	0.48761 904
3	1.00000 000	0.33333 333			
	0.00000 000	1.33333 333			
			8	1.00000 000	0.03571 428
				0.87174 015	0.21070 422
				0.59170 018	0.34112 270
4	1.00000 000	0.16666 667		0.20929 922	0.41245 880
	0.44721 360	0.83333 333			
			9	1.00000 00000	0.02777 77778
				0.89975 79954	0.16549 53616
5	1.00000 000	0.10000 000		0.67718 62795	0.27453 87126
	0.65465 367	0.54444 444		0.36311 74638	0.34642 85110
	0.00000 000	0.71111 111		0.00000 00000	0.37151 92744
			10	1.00000 00000	0.02222 22222
				0.91953 39082	0.13330 59908
6	1.00000 000	0.06666 667		0.73877 38651	0.22488 93420
	0.76505 532	0.37847 496		0.47792 49498	0.29204 26836
	0.28523 152	0.55485 838		0.16527 89577	0.32753 97612

Compiled from Z. Kopal, Numerical analysis, John Wiley & Sons, Inc., New York, N.Y., 1955 (with permission).

Table 25.7 ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION FOR INTEGRANDS WITH A LOGARITHMIC SINGULARITY

$$\int_0^1 f(x)\ln x\,dx = \sum_{i=1}^{n} w_i f(x_i) + \frac{f^{(2n)}(\xi)}{(2n)!} K_n$$

Abscissas $= x_i$ Weight Factors $= w_i$

n	x_i	$-w_i$	K_n	n	x_i	$-w_i$	K_n	n	x_i	$-w_i$	K_n
2	0.112009	0.718539	0.00285	3	0.063891	0.513405	0.00017	4	0.041448	0.383464	0.00001
	0.602277	0.281461			0.368997	0.391980			0.245275	0.386875	
					0.766880	0.094615			0.556165	0.190435	
									0.848982	0.039225	

Compiled from Berthod-Zaborowski, Le calcul des intégrales de la forme $\int_0^1 f(x)\log x\,dx$. H. Mineur, Techniques de calcul numérique, pp. 555-556. Librairie Polytechnique Ch. Béranger, Paris, France, 1952 (with permission).

*See page 11.