

Table 25.2

COEFFICIENTS FOR DIFFERENTIATION

$$\text{Differentiation Formula: } \left. \frac{dkf(x)}{dx^k} \right|_{x=x_j} \approx \frac{k!}{m!h^k} \sum_{i=0}^m A_i f(x_i)$$

FIRST DERIVATIVE (k=1)								THIRD DERIVATIVE (k=3)								
* j	A <sub>0</sub>	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	$\frac{h^k}{k!}$ Error	j	A <sub>0</sub>	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	$\frac{h^k}{k!}$ Error	*
<b>Three Point (m=2)</b>								<b>Four Point (m=3)</b>								
0	-3	4	-1				1/3	0	-1	3	-3	1			-1/4	
1	-1	0	1				-1/6 h <sup>3</sup> f <sup>(3)</sup>	1	-1	3	-3	1			-1/12 h <sup>4</sup> f <sup>(4)</sup>	
2	1	-4	3				1/3	2	-1	3	-3	1			1/12 h <sup>4</sup> f <sup>(4)</sup>	
								3	-1	3	-3	1			1/4	
<b>Four Point (m=3)</b>								<b>Five Point (m=4)</b>								
0	-11	18	-9	2			-1/4	0	-10	36	-48	28	-6		7/24	
1	-2	-3	6	-1			1/12 h <sup>4</sup> f <sup>(4)</sup>	1	-6	20	-24	12	-2		1/24	
2	1	-6	3	2			-1/12 h <sup>4</sup> f <sup>(4)</sup>	2	-2	4	0	-4	2		-1/24 h <sup>5</sup> f <sup>(5)</sup>	
3	-2	9	-18	11			1/4	3	2	-12	24	-20	6		1/24	
								4	6	-28	48	-36	10		7/24	
<b>Five Point (m=4)</b>								<b>Six Point (m=5)</b>								
0	-50	96	-72	32	-6		1/5	0	-85	355	-590	490	-205	35	-5/16	
1	-6	-20	36	-12	2		-1/20	1	-35	125	-170	110	-35	5	-1/48	
2	2	-16	0	16	-2		1/30 h <sup>5</sup> f <sup>(5)</sup>	2	-5	-5	50	-70	35	-5	1/48 h <sup>6</sup> f <sup>(6)</sup>	
3	-2	12	-36	20	6		-1/20	3	5	-35	70	-50	5	5	-1/48 h <sup>6</sup> f <sup>(6)</sup>	
4	6	-32	72	-96	50		1/5	4	-5	35	-110	170	-125	35	1/48	
								5	-35	205	-490	590	-355	85	5/16	
<b>Six Point (m=5)</b>								<b>FOURTH DERIVATIVE (k=4)</b>								
0	-274	600	-600	400	-150	24	-1/6	j	A <sub>0</sub>	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	$\frac{h^k}{k!}$ Error	*
1	-24	-130	240	-120	40	-6	1/30	<b>Five Point (m=4)</b>								
2	6	-60	-40	120	-30	4	-1/60 h <sup>6</sup> f <sup>(6)</sup>	0	1	-4	6	-4	1		-1/12 h <sup>5</sup> f <sup>(5)</sup>	
3	-4	30	-120	40	60	-6	1/60 h <sup>6</sup> f <sup>(6)</sup>	1	1	-4	6	-4	1		-1/24 h <sup>6</sup> f <sup>(6)</sup>	
4	6	-40	120	-240	130	24	-1/30	2	1	-4	6	-4	1		-1/144 h <sup>6</sup> f <sup>(6)</sup>	
5	-24	150	-400	600	-600	274	1/6	3	1	-4	6	-4	1		1/24 h <sup>5</sup> f <sup>(5)</sup>	
								4	1	-4	6	-4	1		1/12 h <sup>5</sup> f <sup>(5)</sup>	
<b>SECOND DERIVATIVE (k=2)</b>								<b>Six Point (m=5)</b>								
* j	A <sub>0</sub>	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	$\frac{h^k}{k!}$ Error	0	15	-70	130	-120	55	-10	17/144	
<b>Three Point (m=2)</b>								<b>Five Point (m=4)</b>								
0	1	-2	1				-1/2 h <sup>3</sup> f <sup>(3)</sup>	1	10	-45	80	-70	30	-5	5/144	
1	1	-2	1				-1/24 h <sup>4</sup> f <sup>(4)</sup>	2	5	-20	30	-20	5	0	-1/144 h <sup>6</sup> f <sup>(6)</sup>	
2	1	-2	1				1/2 h <sup>3</sup> f <sup>(3)</sup>	3	0	5	-20	30	-20	5	-1/144 h <sup>6</sup> f <sup>(6)</sup>	
								4	-5	30	-70	80	-45	10	5/144	
<b>Four Point (m=3)</b>								<b>Six Point (m=5)</b>								
0	6	-15	12	-3			11/24	5	-10	55	-120	130	-70	15	17/144	
1	3	-6	3	0			-1/24 h <sup>4</sup> f <sup>(4)</sup>	<b>FIFTH DERIVATIVE (k=5)</b>								
2	0	3	-6	3			-1/24 h <sup>4</sup> f <sup>(4)</sup>	<b>Six Point (m=5)</b>								
3	-3	12	-15	6			11/24	0	-1	5	-10	10	-5	1	-1/48	
								1	-1	5	-10	10	-5	1	-1/80	
<b>Five Point (m=4)</b>								<b>Six Point (m=5)</b>								
0	35	-104	114	-56	11		-5/12 h <sup>5</sup> f <sup>(5)</sup>	2	-1	5	-10	10	-5	1	-1/240 h <sup>6</sup> f <sup>(6)</sup>	
1	11	-20	6	4	-1		1/24	3	-1	5	-10	10	-5	1	1/240 h <sup>6</sup> f <sup>(6)</sup>	
2	-1	16	-30	16	-1		1/180 h <sup>6</sup> f <sup>(6)</sup>	4	-1	5	-10	10	-5	1	1/80	
3	-1	4	6	-20	11		-1/24 h <sup>5</sup> f <sup>(5)</sup>	5	-1	5	-10	10	-5	1	1/48	
4	11	-56	114	-104	35		5/12									
<b>Six Point (m=5)</b>								<b>Six Point (m=5)</b>								
0	225	-770	1070	-780	305	-50	137/360	0	-1	5	-10	10	-5	1	-1/48	
1	50	-75	-20	70	-30	5	-13/360	1	-1	5	-10	10	-5	1	-1/80	
2	-5	80	-150	80	-5	0	1/180	2	-1	5	-10	10	-5	1	-1/240 h <sup>6</sup> f <sup>(6)</sup>	
3	0	-5	80	-150	80	-5	1/180 h <sup>6</sup> f <sup>(6)</sup>	3	-1	5	-10	10	-5	1	1/240 h <sup>6</sup> f <sup>(6)</sup>	
4	5	-30	70	-20	-75	50	-13/360	4	-1	5	-10	10	-5	1	1/80	
5	-50	305	-780	1070	-770	225	137/360	5	-1	5	-10	10	-5	1	1/48	

Compiled from W. G. Bickley, Formulae for numerical differentiation, Math. Gaz. 25, 19-27, 1941 (with permission).

\*See page II.