25.5.14

$$\text{P:}\quad y_{n+1}=y_{n-5}+\frac{3h}{10}(11y'_n-14y'_{n-1}+26y'_{n-2}-14y'_{n-3}+11y'_{n-4})+O(h^7)$$

$$\text{C:}\quad y_{n+1}=y_{n-3}+\frac{2h}{45}(7y'_{n+1}+32y'_n+12y'_{n-1}+32y'_{n-2}+7y'_{n-3})+O(h^7)$$

Formulas Using Higher Derivatives

25.5.15

$$\text{P:}\quad y_{n+1}=y_{n-2}+3(y_n-y_{n-1})+h^2(y''_n-y''_{n-1})+O(h^5)$$

$$\text{C:}\quad y_{n+1}=y_n+\frac{h}{2}(y'_{n+1}+y'_n)-\frac{h^2}{12}(y''_{n+1}-y''_n)+O(h^5)$$

25.5.16

$$\text{P:}\quad y_{n+1}=y_{n-2}+3(y_n-y_{n-1})+\frac{h^3}{2}(y'''_n+y'''_{n-1})+O(h^7)$$

$$\text{C:}\quad y_{n+1}=y_n+\frac{h}{2}(y'_{n+1}+y'_n)-\frac{h^2}{10}(y''_{n+1}-y''_n)+\frac{h^3}{120}(y'''_{n+1}+y'''_n)+O(h^7)$$

Systems of Differential Equations

First Order: $y'=f(x,y,z)$, $z'=g(x,y,z)$.

Second Order Runge-Kutta

25.5.17

$$y_{n+1}=y_n+\frac{1}{2}(k_1+k_2)+O(h^3),\qquad z_{n+1}=z_n+\frac{1}{2}(l_1+l_2)+O(h^3)$$

$$k_1=hf(x_n,y_n,z_n),\qquad l_1=hg(x_n,y_n,z_n)$$

$$k_2=hf(x_n+h,y_n+k_1,z_n+l_1),\qquad l_2=hg(x_n+h,y_n+k_1,z_n+l_1)$$

Fourth Order Runge-Kutta

25.5.18

$$y_{n+1}=y_n+\frac{1}{6}(k_1+2k_2+2k_3+k_4)+O(h^5),\qquad z_{n+1}=z_n+\frac{1}{6}(l_1+2l_2+2l_3+l_4)+O(h^5)$$

$$k_1=hf(x_n,y_n,z_n)\qquad l_1=hg(x_n,y_n,z_n)$$

$$k_2=hf\left(x_n+\frac{1}{2}h,y_n+\frac{1}{2}k_1,z_n+\frac{1}{2}l_1\right)\qquad l_2=hg\left(x_n+\frac{h}{2},y_n+\frac{k_1}{2},z_n+\frac{l_1}{2}\right)$$

$$k_3=hf\left(x_n+\frac{1}{2}h,y_n+\frac{1}{2}k_2,z_n+\frac{1}{2}l_2\right)\qquad l_3=hg\left(x_n+\frac{h}{2},y_n+\frac{k_2}{2},z_n+\frac{l_2}{2}\right)$$

$$k_4=hf(x_n+h,y_n+k_3,z_n+l_3)\qquad l_4=hg(x_n+h,y_n+k_3,z_n+l_3)$$

Second Order: $y''=f(x,y,y')$

Milne's Method

25.5.19

$$\text{P:}\quad y'_{n+1}=y'_{n-3}+\frac{4h}{3}(2y''_{n-2}-y''_{n-1}+2y''_n)+O(h^5)$$

$$\text{C:}\quad y'_{n+1}=y'_{n-1}+\frac{h}{3}(y''_{n-1}+4y''_n+y''_{n+1})+O(h^5)$$

Runge-Kutta Method

25.5.20

$$y_{n+1}=y_n+h\left[y'_n+\frac{1}{6}(k_1+k_2+k_3)\right]+O(h^5)$$

$$y'_{n+1}=y'_n+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$k_1=hf(x_n,y_n,y'_n)$$

$$k_2=hf\left(x_n+\frac{1}{2}h,y_n+\frac{h}{2}y'_n+\frac{h}{8}k_1,y'_n+\frac{k_1}{2}\right)$$

$$k_3=hf\left(x_n+\frac{1}{2}h,y_n+\frac{h}{2}y'_n+\frac{h}{8}k_1,y'_n+\frac{k_2}{2}\right)\quad *$$

$$k_4=hf\left(x_n+h,y_n+hy'_n+\frac{h}{2}k_3,y'_n+k_3\right)$$

Second Order: $y''=f(x,y)$

Milne's Method

25.5.21

$$\text{P:}\quad y_{n+1}=y_n+y_{n-2}-y_{n-3}+\frac{h^2}{4}(5y''_n+2y''_{n-1}+5y''_{n-2})+O(h^6)$$

$$\text{C:}\quad y_n=2y_{n-1}-y_{n-2}+\frac{h^2}{12}(y''_n+10y''_{n-1}+y''_{n-2})+O(h^6)$$

Runge-Kutta Method

25.5.22

$$y_{n+1}=y_n+h\left(y'_n+\frac{1}{6}(k_1+2k_2)\right)+O(h^4)$$

$$y'_{n+1}=y'_n+\frac{1}{6}k_1+\frac{2}{3}k_2+\frac{1}{6}k_3$$

$$k_1=hf(x_n,y_n)$$

$$k_2=hf\left(x_n+\frac{h}{2},y_n+\frac{h}{2}y'_n+\frac{h}{8}k_1\right)$$

$$k_3=hf\left(x_n+h,y_n+hy'_n+\frac{h}{2}k_2\right).$$

*See page II.