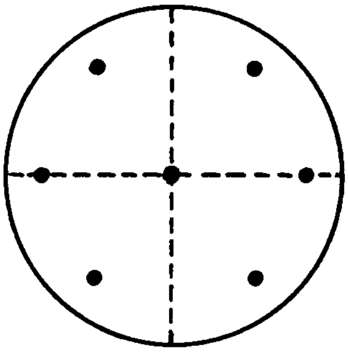
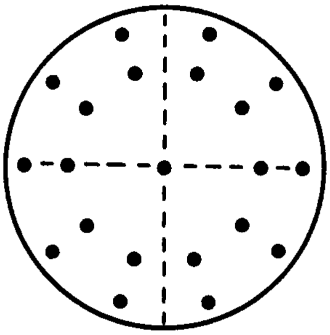
(x_i, y_i)	w_i	
$(0,0)$	1/6	
$(\pm h, 0)$	1/24	$R=O(h^6)$
$(0, \pm h)$	1/24	
$\left(\pm\frac{h}{2}, \pm\frac{h}{2}\right)$	1/6	



(x_i, y_i)	w_i	
$(0,0)$	1/4	
$\left(\pm\sqrt{\frac{2}{3}}\,h, 0\right)$	1/8	$R=O(h^6)$
$\left(\pm\sqrt{\frac{1}{6}}\,h, \pm\frac{h}{2}\sqrt{2}\right)$	1/8	



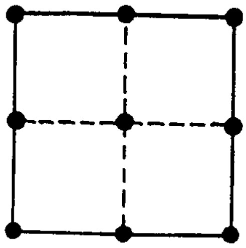
(x_i, y_i)	w_i
$(0,0)$	1/9
$\left(\sqrt{\frac{6-\sqrt{6}}{10}}\,h\cos\frac{2\pi k}{10}, \sqrt{\frac{6-\sqrt{6}}{10}}\,h\sin\frac{2\pi k}{10}\right)$	$\frac{16+\sqrt{6}}{360}$
$(k=1, \ldots, 10)$	
$\left(\sqrt{\frac{6+\sqrt{6}}{10}}\,h\cos\frac{2\pi k}{10}, \sqrt{\frac{6+\sqrt{6}}{10}}\,h\sin\frac{2\pi k}{10}\right)$	$\frac{16-\sqrt{6}}{360}$

$R=O(h^{10})$

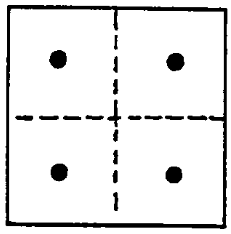
Square[4] S: $|x|\leq h, |y|\leq h$

25.4.62

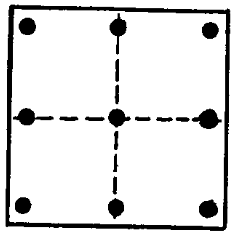
$$\frac{1}{4h^2}\iint_S f(x,y)\,dx\,dy=\sum_{i=1}^{n} w_i f(x_i, y_i)+R$$



(x_i, y_i)	w_i	
$(0,0)$	4/9	
$(\pm h, \pm h)$	1/36	$R=O(h^4)$
$(\pm h, 0)$	1/9	
$(0, \pm h)$	1/9	



(x_i, y_i)	w_i	
$\left(\pm h\sqrt{\frac{1}{3}}, \pm h\sqrt{\frac{1}{3}}\right)$	1/4	$R=O(h^4)$



(x_i, y_i)	w_i
$(0,0)$	16/81

[4] For regions, such as the square, cube, cylinder, etc., which are the Cartesian products of lower dimensional regions, one may always develop integration rules by "multiplying together" the lower dimensional rules. Thus if

$$\int_0^1 f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

is a one dimensional rule, then

$$\int_0^1\int_0^1 f(x,y)\,dx\,dy \approx \sum_{i,j=1}^{n} w_i w_j f(x_i, x_j)$$

becomes a two dimensional rule. Such rules are not necessarily the most "economical".