

25.4.56 $S_{2n-1} = \sum_{i=1}^n f_{2i-1} \sin (tx_{2i-1})$

25.4.57 $C'_{2n-1} = \sum_{i=1}^n f_{2i-1}^{(3)} \cos (tx_{2i-1})$

(See Table 25.11 for α, β, γ .)

Iterated Integrals

25.4.58

$$\int_0^x dt_n \int_0^{t_n} dt_{n-1} \dots \int_0^{t_3} dt_2 \int_0^{t_2} f(t_1) dt_1$$

$$= \frac{1}{(n-1)!} \int_0^x (x-t)^{n-1} f(t) dt$$

25.4.59

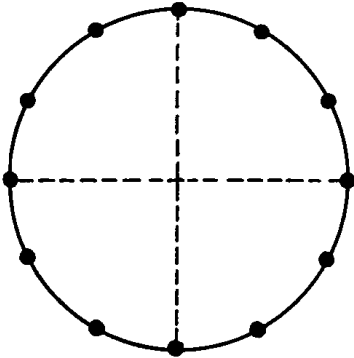
$$\int_a^x dt_n \int_a^{t_n} dt_{n-1} \dots \int_a^{t_3} dt_2 \int_a^{t_2} f(t_1) dt_1$$

$$= \frac{(x-a)^n}{(n-1)!} \int_0^1 t^{n-1} f(x-(x-a)t) dt$$

Multidimensional Integration

Circumference of Circle $\Gamma: x^2 + y^2 = h^2$.

25.4.60

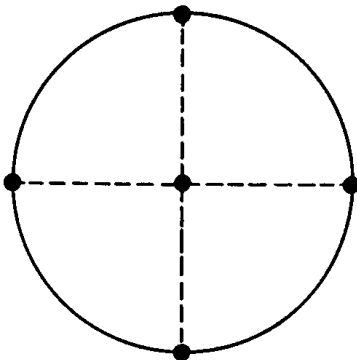


$$\frac{1}{2\pi h} \int_{\Gamma} f(x,y) ds = \frac{1}{2m} \sum_{n=1}^{2m} f\left(h \cos \frac{\pi n}{m}, h \sin \frac{\pi n}{m}\right)$$

$$+ O(h^{2m-2})$$

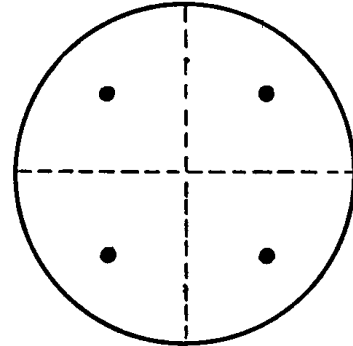
Circle $C: x^2 + y^2 \leq h^2$.

25.4.61

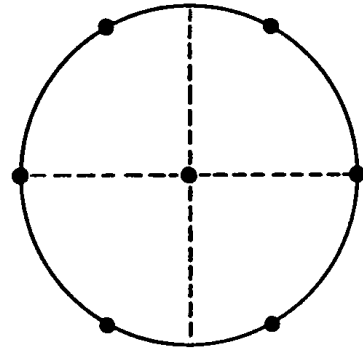


$$\frac{1}{\pi h^2} \iint_C f(x,y) dx dy = \sum_{i=1}^n w_i f(x_i, y_i) + R$$

(x_i, y_i)	w_i	
$(0, 0)$	$1/2$	$R = O(h^4)$
$(\pm h, 0), (0, \pm h)$	$1/8$	



(x_i, y_i)	w_i	
$(\pm \frac{h}{2}, \pm \frac{h}{2})$	$1/4$	$R = O(h^4)$



(x_i, y_i)	w_i	
$(0, 0)$	$1/2$	
$(\pm h, 0)$	$1/12$	$R = O(h^4)$
$(\pm \frac{h}{2}, \pm \frac{h}{2} \sqrt{3})$	$1/12$	

