

Remainder:

$$R_n = \frac{\pi}{(2n)! 2^{4n+1}} f^{(2n)}(\xi) \quad (0 < \xi < 1)$$

25.4.43

$$\int_a^b f(x) \sqrt{\frac{x-a}{b-x}} dx = (b-a) \sum_{i=1}^n w_i f(y_i) + R_n$$

$$y_i = a + (b-a)x_i$$

Related orthogonal polynomials:

$$\frac{1}{\sqrt{x}} T_{2n+1}(\sqrt{x})$$

Abscissas:

$$x_i = \cos^2 \frac{2i-1}{2n+1} \cdot \frac{\pi}{2}$$

Weights:

$$w_i = \frac{2\pi}{2n+1} x_i$$

25.4.44 $\int_0^1 \ln x f(x) dx = \sum_{i=1}^n w_i f(x_i) + R_n$

Related orthogonal polynomials: polynomials orthogonal with respect to the weight function $-\ln x$

Abscissas: See Table 25.7

Weights: See Table 25.7

25.4.45

$$\int_0^\infty e^{-x} f(x) dx = \sum_{i=1}^n w_i f(x_i) + R_n$$

Related orthogonal polynomials: Laguerre polynomials $L_n(x)$.

Abscissas: x_i is the i^{th} zero of $L_n(x)$

Weights:

$$* \quad w_i = \frac{x_i}{(n+1)^2 [L_{n+1}(x_i)]^2}$$

(See Table 25.9 for x_i and w_i .)

Remainder:

$$R_n = \frac{(n!)^2}{(2n)!} f^{(2n)}(\xi) \quad (0 < \xi < \infty)$$

25.4.46

$$\int_{-\infty}^\infty e^{-x^2} f(x) dx = \sum_{i=1}^n w_i f(x_i) + R_n$$

Related orthogonal polynomials: Hermite polynomials $H_n(x)$.

Abscissas: x_i is the i^{th} zero of $H_n(x)$

Weights:

$$\frac{2^{n-1} n! \sqrt{\pi}}{n^2 [H_{n-1}(x_i)]^2}$$

(See Table 25.10 for x_i and w_i .)

Remainder:

$$R_n = \frac{n! \sqrt{\pi}}{2^n (2n)!} f^{(2n)}(\xi) \quad (-\infty < \xi < \infty)$$

Filon's Integration Formula³

25.4.47

$$\int_{x_0}^{x_{2n}} f(x) \cos tx dx = h \left[\alpha(th) (f_{2n} \sin tx_{2n} - f_0 \sin tx_0) + \beta(th) \cdot C_{2n} + \gamma(th) \cdot C_{2n-1} + \frac{2}{45} th^4 S'_{2n-1} \right] - R_n$$

25.4.48

$$C_{2n} = \sum_{i=0}^n f_{2i} \cos(tx_{2i}) - \frac{1}{2} [f_{2n} \cos tx_{2n} + f_0 \cos tx_0]$$

25.4.49

$$C_{2n-1} = \sum_{i=1}^n f_{2i-1} \cos tx_{2i-1}$$

25.4.50

$$S'_{2n-1} = \sum_{i=1}^n f_{2i-1}^{(3)} \sin tx_{2i-1}$$

25.4.51

$$R_n = \frac{1}{90} nh^5 f^{(4)}(\xi) + O(th^7)$$

25.4.52

$$\alpha(\theta) = \frac{1}{\theta} + \frac{\sin 2\theta}{2\theta^2} - \frac{2 \sin^2 \theta}{\theta^3}$$

$$\beta(\theta) = 2 \left(\frac{1 + \cos^2 \theta}{\theta^2} - \frac{\sin 2\theta}{\theta^3} \right)$$

$$\gamma(\theta) = 4 \left(\frac{\sin \theta}{\theta^3} - \frac{\cos \theta}{\theta^2} \right)$$

For small θ we have

25.4.53

$$\alpha = \frac{2\theta^3}{45} - \frac{2\theta^5}{315} + \frac{2\theta^7}{4725} - \dots$$

$$\beta = \frac{2}{3} + \frac{2\theta^2}{15} - \frac{4\theta^4}{105} + \frac{2\theta^6}{567} - \dots$$

$$\gamma = \frac{4}{3} - \frac{2\theta^2}{15} + \frac{\theta^4}{210} - \frac{\theta^6}{11340} + \dots$$

25.4.54

$$\int_{x_0}^{x_{2n}} f(x) \sin tx dx = h \left[\alpha(th) (f_0 \cos tx_0 - f_{2n} \cos tx_{2n}) + \beta S_{2n} + \gamma S_{2n-1} + \frac{2}{45} th^4 C'_{2n-1} \right] - R_n$$

25.4.55

$$S_{2n} = \sum_{i=0}^n f_{2i} \sin(tx_{2i}) - \frac{1}{2} [f_{2n} \sin(tx_{2n}) + f_0 \sin(tx_0)]$$

³ For certain difficulties associated with this formula, see the article by J. W. Tukey, p. 400, "On Numerical Approximation," Ed. R. E. Langer, Madison, 1959.

*See page 11.