25.4.20

$$\int_{x_0}^{x_{10}} f(x)dx=\frac{5h}{299376}\{16067(f_0+f_{10})+106300(f_1+f_9)-48525(f_2+f_8)+272400(f_3+f_7)-260550(f_4+f_6)+427368f_5\}-\frac{1346350}{326918592}f^{(12)}(\xi)h^{13}$$

Newton-Cotes Formulas (Open Type)

25.4.21

$$\int_{x_0}^{x_3} f(x)dx=\frac{3h}{2}(f_1+f_2)+\frac{f^{(2)}(\xi)h^3}{4}$$

25.4.22

$$\int_{x_0}^{x_4} f(x)dx=\frac{4h}{3}(2f_1-f_2+2f_3)+\frac{28f^{(4)}(\xi)h^5}{90}$$

25.4.23

$$\int_{x_0}^{x_5} f(x)dx=\frac{5h}{24}(11f_1+f_2+f_3+11f_4)+\frac{95f^{(4)}(\xi)h^5}{144}$$

25.4.24

$$\int_{x_0}^{x_6} f(x)dx=\frac{6h}{20}(11f_1-14f_2+26f_3-14f_4+11f_5)+\frac{41f^{(6)}(\xi)h^7}{140}$$

25.4.25

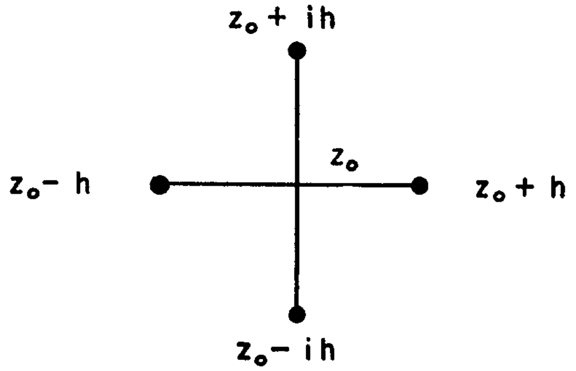
$$\int_{x_0}^{x_7} f(x)dx=\frac{7h}{1440}(611f_1-453f_2+562f_3+562f_4-453f_5+611f_6)+\frac{5257}{8640}f^{(6)}(\xi)h^7$$

25.4.26

$$\int_{x_0}^{x_8} f(x)dx=\frac{8h}{945}(460f_1-954f_2+2196f_3-2459f_4+2196f_5-954f_6+460f_7)+\frac{3956}{14175}f^{(8)}(\xi)h^9$$

Five Point Rule for Analytic Functions

25.4.27



$$\int_{z_0-h}^{z_0+h} f(z)dz=\frac{h}{15}\{24f(z_0)+4[f(z_0+h)+f(z_0-h)]-[f(z_0+ih)+f(z_0-ih)]\}+R$$

$|R|\leq\frac{|h|^7}{1890}\operatorname*{Max}_{z\epsilon S}|f^{(6)}(z)|$, S designates the square with vertices $z_0+i^kh(k=0,1,2,3)$; h can be complex.

Chebyshev's Equal Weight Integration Formula

25.4.28 $$\int_{-1}^{1} f(x)dx=\frac{2}{n}\sum_{i=1}^{n} f(x_i)+R_n$$

Abscissas: x_i is the i^{th} zero of the polynomial part of

$$x^n\exp\left[\frac{-n}{2\cdot 3x^2}-\frac{n}{4\cdot 5x^3}-\frac{n}{6\cdot 7x^4}-\ldots\right]$$

(See **Table 25.5** for x_i.)

For $n=8$ and $n\geq 10$ some of the zeros are complex.

Remainder:

$$R_n=\int_{-1}^{+1}\frac{x^{n+1}}{(n+1)!}f^{(n+1)}(\xi)dx-\frac{2}{n(n+1)!}\sum_{i=1}^{n}x_i^{n+1}f^{(n+1)}(\xi_i)$$

where $\xi=\xi(x)$ satisfies $0\leq\xi\leq x$ and $0\leq\xi_i\leq x_i$

$$(i=1,\ldots,n)$$

Integration Formulas of Gaussian Type

(For Orthogonal Polynomials see chapter **22**)

Gauss' Formula

25.4.29 $$\int_{-1}^{1} f(x)dx=\sum_{i=1}^{n} w_if(x_i)+R_n$$

Related orthogonal polynomials: Legendre polynomials $P_n(x)$, $P_n(1)=1$

Abscissas: x_i is the i^{th} zero of $P_n(x)$

Weights: $w_i=2/(1-x_i^2)[P_n'(x_i)]^2$
(See **Table 25.4** for x_i and w_i.)

$$R_n=\frac{2^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3}f^{(2n)}(\xi)\qquad(-1<\xi<1)$$

Gauss' Formula, Arbitrary Interval

25.4.30 $$\int_a^b f(y)dy=\frac{b-a}{2}\sum_{i=1}^{n} w_if(y_i)+R_n$$

$$y_i=\left(\frac{b-a}{2}\right)x_i+\left(\frac{b+a}{2}\right)$$

*See page II.