

25.3.3

$$R'_n(x) = \frac{f^{(n+1)}}{(n+1)!} (\xi) \pi'_n(x) + \frac{\pi_n(x)}{(n+1)!} \frac{d}{dx} f^{(n+1)}(\xi)$$

$$\xi = \xi(x) \quad (x_0 < \xi < x_n)$$

Equally Spaced Abscissas

Three Points

25.3.4

$$f'_p = f'(x_0 + ph)$$

$$= \frac{1}{h} \left\{ (p - \frac{1}{2}) f_{-1} - 2p f_0 + (p + \frac{1}{2}) f_1 \right\} + R'_2$$

Four Points

25.3.5

$$f'_p = f'(x_0 + ph) = \frac{1}{h} \left\{ -\frac{3p^2 - 6p + 2}{6} f_{-1} \right.$$

$$+ \frac{3p^2 - 4p - 1}{2} f_0 - \frac{3p^2 - 2p - 2}{2} f_1$$

$$\left. + \frac{3p^2 - 1}{6} f_2 \right\} + R'_3$$

Five Points

25.3.6

$$f'_p = f'(x_0 + ph) = \frac{1}{h} \left\{ \frac{2p^3 - 3p^2 - p + 1}{12} f_{-2} \right.$$

$$- \frac{4p^3 - 3p^2 - 8p + 4}{6} f_{-1} + \frac{2p^3 - 5p}{2} f_0$$

$$- \frac{4p^3 + 3p^2 - 8p - 4}{6} f_1$$

$$\left. + \frac{2p^3 + 3p^2 - p - 1}{12} f_2 \right\} + R'_4$$

For numerical values of differentiation coefficients see Table 25.2.

Markoff's Formulas

(Newton's Forward Difference Formula Differentiated)

25.3.7

$$f'(a_0 + ph) = \frac{1}{h} \left[\Delta_0 + \frac{2p-1}{2} \Delta_0^2 \right.$$

$$\left. + \frac{3p^2 - 6p + 2}{6} \Delta_0^3 + \dots + \frac{d}{dp} \binom{p}{n} \Delta_0^n \right] + R'_n$$

25.3.8

$$R'_n = h^n f^{(n+1)}(\xi) \frac{d}{dp} \binom{p}{n+1} + h^{n+1} \binom{p}{n+1} \frac{d}{dx} f^{(n+1)}(\xi)$$

$$(a_0 < \xi < a_n)$$

25.3.9 $hf'_0 = \Delta_0 - \frac{1}{2} \Delta_0^2 + \frac{1}{3} \Delta_0^3 - \frac{1}{4} \Delta_0^4 + \dots$

25.3.10 $h^2 f_0^{(2)} = \Delta_0^2 - \Delta_0^3 + \frac{11}{12} \Delta_0^4 - \frac{5}{6} \Delta_0^5 + \dots$

25.3.11

$$h^3 f_0^{(3)} = \Delta_0^3 - \frac{3}{2} \Delta_0^4 + \frac{7}{4} \Delta_0^5 - \frac{15}{8} \Delta_0^6 + \dots$$

25.3.12

$$h^4 f_0^{(4)} = \Delta_0^4 - 2\Delta_0^5 + \frac{17}{6} \Delta_0^6 - \frac{7}{2} \Delta_0^7 + \dots$$

25.3.13

$$h^5 f_0^{(5)} = \Delta_0^5 - \frac{5}{2} \Delta_0^6 + \frac{25}{6} \Delta_0^7 - \frac{35}{6} \Delta_0^8 + \dots$$

Everett's Formula

25.3.14

$$hf'(x_0 + ph) \approx -f_0 + f_1 - \frac{3p^2 - 6p + 2}{6} \delta_0^2 + \frac{3p^2 - 1}{6} \delta_1^2$$

$$- \frac{5p^4 - 20p^3 + 15p^2 + 10p - 6}{120} \delta_0^4 + \frac{5p^4 - 15p^2 + 4}{120} \delta_1^4$$

$$+ \dots - \left[\binom{p+n-1}{2n+1} \right]' \delta_0^{2n} + \left[\binom{p+n}{2n+1} \right]' \delta_1^{2n}$$

25.3.15

$$hf'_0 \approx -f_0 + f_1 - \frac{1}{3} \delta_0^2 - \frac{1}{6} \delta_1^2 + \frac{1}{20} \delta_0^4 + \frac{1}{30} \delta_1^4$$

Differences in Terms of Derivatives

25.3.16

$$\Delta_0 \approx hf'_0 + \frac{h^2}{2!} f_0^{(2)} + \frac{h^3}{3!} f_0^{(3)} + \frac{h^4}{4!} f_0^{(4)} + \frac{h^5}{5!} f_0^{(5)}$$

25.3.17

$$\Delta_0^2 \approx h^2 f_0^{(2)} + h^3 f_0^{(3)} + \frac{7}{12} h^4 f_0^{(4)} + \frac{1}{4} h^5 f_0^{(5)}$$

25.3.18

$$\Delta_0^3 \approx h^3 f_0^{(3)} + \frac{3}{2} h^4 f_0^{(4)} + \frac{5}{4} h^5 f_0^{(5)}$$

25.3.19

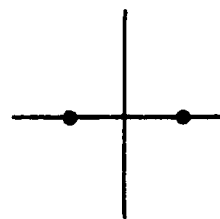
$$\Delta_0^4 \approx h^4 f_0^{(4)} + 2h^5 f_0^{(5)}$$

25.3.20

$$\Delta_0^5 \approx h^5 f_0^{(5)}$$

Partial Derivatives

25.3.21



$$\frac{\partial f_{0,0}}{\partial x} = \frac{1}{2h} (f_{1,0} - f_{-1,0}) + O(h^2)$$