Simultaneous Throwback

25.2.43

$$f(x_0+ph)=(1-p)f_0+pf_1+E_2\delta^2_{m,0}+F_2\delta^2_{m,1}+E_4\delta^4_{m,0}+F_4\delta^4_{m,1}+R$$

25.2.44 $$\delta^2_m=\delta^2-.01312\delta^6+.0043\delta^8-.001\delta^{10}$$

25.2.45 $$\delta^4_m=\delta^4-.27827\delta^6+.0685\delta^8-.016\delta^{10}$$

25.2.46 $$R\approx .00000083|\mu\delta^6_{\frac{1}{2}}|+.0000094\delta^7$$

Bessel's Formula With Throwback

25.2.47

$$f(x_0+ph)=(1-p)f_0+pf_1+B_2(\delta^2_{m,0}+\delta^2_{m,1})+B_3\delta^3_{\frac{1}{2}}+R,\ B_2=\frac{p(p-1)}{4},\ B_3=\frac{p(p-1)\,(p-\frac{1}{2})}{6}$$

25.2.48 $$\delta^2_m=\delta^2-.184\delta^4$$

25.2.49 $$R\approx .00045|\mu\delta^4_{\frac{1}{2}}|+.00087|\delta^5_{\frac{1}{2}}|$$

Thiele's Interpolation Formula

25.2.50

$$f(x)=f(x_1)+\cfrac{x-x_1}{\rho(x_1,x_2)+\cfrac{x-x_2}{\rho_2(x_1,x_2,x_3)-f(x_1)+\cfrac{x-x_3}{\left(\begin{array}{l}\rho_3(x_1,x_2,x_3,x_4)\\ \quad-\rho(x_1,x_2)+\dots\end{array}\right)}}}$$

(For reciprocal differences, ρ, see **25.1.12.**)

Trigonometric Interpolation

Gauss' Formula

25.2.51 $$f(x)\approx\sum_{k=0}^{2n} f_k\zeta_k(x)=t_n(x)$$

25.2.52

$$\zeta_k(x)=\frac{\sin\frac{1}{2}(x-x_0)\ \dots\ \sin\frac{1}{2}(x-x_{k-1})}{\sin\frac{1}{2}(x_k-x_0)\ \dots\ \sin\frac{1}{2}(x_k-x_{k-1})}\cdot\frac{\sin\frac{1}{2}(x-x_{k+1})\ \dots\ \sin\frac{1}{2}(x-x_{2n})}{\sin\frac{1}{2}(x_k-x_{k+1})\ \dots\ \sin\frac{1}{2}(x_k-x_{2n})}$$

$t_n(x)$ is a trigonometric polynomial of degree n such that $t_n(x_k)=f_k$ $\quad(k=0,1,\ .\ .\ .,2n)$

Harmonic Analysis

Equally spaced abscissas

$$x_0=0,\qquad x_1,\ .\ .\ .,x_{m-1},x_m=2\pi$$

25.2.53

$$f(x)\approx\frac{1}{2}a_0+\sum_{k=1}^{n}(a_k\cos kx+b_k\sin kx)$$

25.2.54 $$m=2n+1$$

$$a_k=\frac{2}{2n+1}\sum_{r=0}^{2n} f_r\cos kx_r;\qquad b_k=\frac{2}{2n+1}\sum_{r=0}^{2n} f_r\sin kx_r$$

$$(k=0,1,\ .\ .\ .,n)$$

25.2.55 $$m=2n$$

$$a_k=\frac{1}{n}\sum_{r=0}^{2n-1} f_r\cos kx_r;\qquad b_k=\frac{1}{n}\sum_{r=0}^{2n-1} f_r\sin kx_r$$

$(k=0,1,\ .\ .\ .,n)\qquad(k=0,1,\ .\ .\ .,n-1)$

b_n is arbitrary.

Subtabulation

Let $f(x)$ be tabulated initially in intervals of width h. It is desired to subtabulate $f(x)$ in intervals of width h/m. Let Δ and $\overline{\Delta}$ designate differences with respect to the original and the final intervals respectively. Thus $\overline{\Delta}_0=f\left(x_0+\frac{h}{m}\right)-f(x_0)$. Assuming that the original 5th order differences are zero,

25.2.56

$$\overline{\Delta}_0=\frac{1}{m}\Delta_0+\frac{1-m}{2m^2}\Delta^2_0+\frac{(1-m)(1-2m)}{6m^3}\Delta^3_0+\frac{(1-m)(1-2m)(1-3m)}{24m^4}\Delta^4_0$$

$$\overline{\Delta}^2_0=\frac{1}{m^2}\Delta^2_0+\frac{1-m}{m^3}\Delta^3_0+\frac{(1-m)(7-11m)}{12m^4}\Delta^4_0$$

$$\overline{\Delta}^3_0=\frac{1}{m^3}\Delta^3_0+\frac{3(1-m)}{2m^4}\Delta^4_0$$

$$\overline{\Delta}^4_0=\frac{1}{m^4}\Delta^4_0$$

From this information we may construct the final tabulation by addition. For $m=10$,

25.2.57

$$\overline{\Delta}_0=.1\Delta_0-.045\Delta^2_0+.0285\Delta^3_0-.02066\Delta^4_0$$
$$\overline{\Delta}^2_0=.01\Delta^2_0-.009\Delta^3_0+.007725\Delta^4_0$$
$$\overline{\Delta}^3_0=.001\Delta^3_0-.00135\Delta^4_0$$
$$\overline{\Delta}^4_0=.0001\Delta^4_0$$

Linear Inverse Interpolation

Find p, given $f_p(=f(x_0+ph))$.

Linear

25.2.58 $$p\approx\frac{f_p-f_0}{f_1-f_0}$$