Taylor Expansion

25.2.24

$$f(x)=f_0+(x-x_0)f_0'+\frac{(x-x_0)^2}{2!}f_0^{(2)}+\ldots+\frac{(x-x_0)^n}{n!}f_0^{(n)}+R_n$$

25.2.25
$$R_n=\int_{x_0}^{x}f^{(n+1)}(t)\,\frac{(x-t)^n}{n!}\,dt=\frac{(x-x_0)^{n+1}}{(n+1)!}f^{(n+1)}(\xi)\qquad (x_0<\xi<x)$$

Newton's Divided Difference Interpolation Formula

25.2.26

$$f(x)=f_0+\sum_{k=1}^{n}\pi_{k-1}(x)\,[x_0,x_1,\ldots,x_k]+R_n$$

$$\begin{array}{llccc}
x_0 & f_0 & & & \\
 & & [x_0,x_1] & & \\
x_1 & f_1 & & [x_0,x_1,x_2] & \\
 & & [x_1,x_2] & & [x_0,x_1,x_2,x_3] \\
x_2 & f_2 & & [x_1,x_2,x_3] & \\
 & & [x_2,x_3] & & \\
x_3 & f_3 & & &
\end{array}$$

25.2.27

$$R_n(x)=\pi_n(x)\,[x_0,\ldots,x_n,x]=\pi_n(x)\,\frac{f^{(n+1)}(\xi)}{(n+1)!}\qquad (x_0<\xi<x_n)$$

(For π_n see 25.1.6.)

Newton's Forward Difference Formula

25.2.28

$$f(x_0+ph)=f_0+p\Delta_0+\binom{p}{2}\Delta_0^2+\ldots+\binom{p}{n}\Delta_0^n+R_n$$

$$\begin{array}{llccc}
x_0 & f_0 & & & \\
 & & \Delta_0 & & \\
x_1 & f_1 & & \Delta_0^2 & \\
 & & \Delta_1 & & \Delta_0^3 \\
x_2 & f_2 & & \Delta_1^2 & \\
 & & \Delta_2 & & \\
x_3 & f_3 & & &
\end{array}$$

25.2.29

$$R_n=h^{n+1}\binom{p}{n+1}f^{(n+1)}(\xi)\approx\binom{p}{n+1}\Delta_0^{n+1}\qquad (x_0<\xi<x_n)$$

Relation Between Newton and Lagrange Coefficients

25.2.30

$$\binom{p}{2}=A_{-1}^3(p)\qquad \binom{p}{3}=-A_{-1}^4(p)\qquad \binom{p}{4}=A_2^5(1-p)$$

$$\binom{p}{5}=A_3^6(2-p)$$

Everett's Formula

25.2.31

$$\begin{aligned}f(x_0+ph)&=(1-p)f_0+pf_1-\frac{p(p-1)(p-2)}{3!}\delta_0^2\\&\quad+\frac{(p+1)p(p-1)}{3!}\delta_1^2+\ldots-\binom{p+n-1}{2n+1}\delta_0^{2n}\\&\quad+\binom{p+n}{2n+1}\delta_1^{2n}+R_{2n}\\&=(1-p)f_0+pf_1+E_2\delta_0^2+F_2\delta_1^2+E_4\delta_0^4\\&\quad+F_4\delta_1^4+\ldots+R_{2n}\end{aligned}$$

$$\begin{array}{llccc}
x_0 & f_0 & \delta_0^2 & & \delta_0^4 \\
 & & \delta_{\frac{1}{2}} & \delta_{\frac{1}{2}}^3 & \\
x_1 & f_1 & \delta_1^2 & & \delta_1^4
\end{array}$$

25.2.32

$$R_{2n}=h^{2n+2}\binom{p+n}{2n+2}f^{(2n+2)}(\xi)$$

$$\approx\binom{p+n}{2n+2}\left[\frac{\Delta_{-n-1}^{2n+2}+\Delta_{-n}^{2n+2}}{2}\right]\qquad (x_{-n}<\xi<x_{n+1})$$

Relation Between Everett and Lagrange Coefficients

25.2.33

$$E_2=A_{-1}^4\qquad E_4=A_{-2}^6\qquad E_6=A_{-3}^8$$

$$F_2=A_2^4\qquad F_4=A_3^6\qquad F_6=A_4^8$$

Everett's Formula With Throwback (Modified Central Difference)

25.2.34

$$f(x_0+ph)=(1-p)f_0+pf_1+E_2\delta_{m,0}^2+F_2\delta_{m,1}^2+R$$

25.2.35 $\delta_m^2=\delta^2-.184\delta^4$

25.2.36 $R\approx.00045|\mu\delta_{\frac{1}{2}}^4|+.00061|\delta_{\frac{1}{2}}^5|$

25.2.37

$$f(x_0+ph)=(1-p)f_0+pf_1+E_2\delta_0^2+F_2\delta_1^2+E_4\delta_{m,0}^4+F_4\delta_{m,1}^4+R$$

25.2.38 $\delta_m^4=\delta^4-.207\delta^6+\ldots$

25.2.39 $R\approx.000032|\mu\delta_{\frac{1}{2}}^6|+.000052|\delta_{\frac{1}{2}}^7|$

25.2.40

$$f(x_0+ph)=(1-p)f_0+pf_1+E_2\delta_0^2+F_2\delta_1^2+E_4\delta_0^4+F_4\delta_1^4+E_6\delta_{m,0}^6+F_6\delta_{m,1}^6+R$$

25.2.41 $\delta_m^6=\delta^6-.218\delta^8+.049\delta^{10}+\ldots$

25.2.42 $R\approx.0000037|\mu\delta_{\frac{1}{2}}^8|+\ldots$