Series Expansions

4.6.31

$$\operatorname{arcsinh} z = z - \frac{1}{2\cdot 3}z^3 + \frac{1\cdot 3}{2\cdot 4\cdot 5}z^5 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7}z^7 + \cdots \qquad (|z|<1)$$

$$= \ln 2z + \frac{1}{2\cdot 2z^2} - \frac{1\cdot 3}{2\cdot 4\cdot 4z^4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6z^6} - \cdots \qquad (|z|>1)$$

4.6.32

$$\operatorname{arccosh} z = \ln 2z - \frac{1}{2\cdot 2z^2} - \frac{1\cdot 3}{2\cdot 4\cdot 4z^4} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6z^6} - \cdots \qquad (|z|>1)$$

4.6.33 $$\operatorname{arctanh} z = z + \frac{z^3}{3} + \frac{z^5}{5} + \frac{z^7}{7} + \cdots \qquad (|z|<1)$$

4.6.34 $$\operatorname{arccoth} z = \frac{1}{z} + \frac{1}{3z^3} + \frac{1}{5z^5} + \frac{1}{7z^7} + \cdots \qquad (|z|>1)$$

Continued Fractions

4.6.35 $$\operatorname{arctanh} z = \cfrac{z}{1-}\,\cfrac{z^2}{3-}\,\cfrac{4z^2}{5-}\,\cfrac{9z^2}{7-}\cdots$$

(z in the cut plane of Figure 4.7.)

4.6.36

$$\frac{\operatorname{arcsinh} z}{\sqrt{1+z^2}} = \cfrac{z}{1+}\,\cfrac{1\cdot 2z^2}{3+}\,\cfrac{1\cdot 2z^2}{5+}\,\cfrac{3\cdot 4z^2}{7+}\,\cfrac{3\cdot 4z^2}{9+}\cdots$$

Differentiation Formulas

4.6.37 $$\frac{d}{dz}\operatorname{arcsinh} z = (1+z^2)^{-\frac{1}{2}}$$

4.6.38 $$\frac{d}{dz}\operatorname{arccosh} z = (z^2-1)^{-\frac{1}{2}}$$

4.6.39 $$\frac{d}{dz}\operatorname{arctanh} z = (1-z^2)^{-1}$$

4.6.40 $$\frac{d}{dz}\operatorname{arccsch} z = \mp\frac{1}{z(1+z^2)^{\frac{1}{2}}}$$

(according as $\Re z \gtrless 0$)

4.6.41 $$\frac{d}{dz}\operatorname{arcsech} z = \mp\frac{1}{z(1-z^2)^{\frac{1}{2}}}$$

4.6.42 $$\frac{d}{dz}\operatorname{arccoth} z = (1-z^2)^{-1}$$

Integration Formulas

4.6.43 $$\int \operatorname{arcsinh} z\,dz = z\operatorname{arcsinh} z - (1+z^2)^{\frac{1}{2}}$$

4.6.44 $$\int \operatorname{arccosh} z\,dz = z\operatorname{arccosh} z - (z^2-1)^{\frac{1}{2}}$$

4.6.45 $$\int \operatorname{arctanh} z\,dz = z\operatorname{arctanh} z + \tfrac{1}{2}\ln(1-z^2)$$

4.6.46 $$\int \operatorname{arccsch} z\,dz = z\operatorname{arccsch} z \pm \operatorname{arcsinh} z \quad *$$

(according as $\Re z \gtrless 0$)

4.6.47 $$\int \operatorname{arcsech} z\,dz = z\operatorname{arcsech} z \pm \arcsin z \quad *$$

4.6.48 $$\int \operatorname{arccoth} z\,dz = z\operatorname{arccoth} z + \tfrac{1}{2}\ln(z^2-1)$$

4.6.49

$$\int z\operatorname{arcsinh} z\,dz = \frac{2z^2+1}{4}\operatorname{arcsinh} z - \frac{z}{4}(z^2+1)^{\frac{1}{2}}$$

4.6.50

$$\int z^n \operatorname{arcsinh} z\,dz = \frac{z^{n+1}}{n+1}\operatorname{arcsinh} z - \frac{1}{n+1}\int \frac{z^{n+1}}{(1+z^2)^{\frac{1}{2}}}dz \qquad (n\neq -1)$$

4.6.51

$$\int z\operatorname{arccosh} z\,dz = \frac{2z^2-1}{4}\operatorname{arccosh} z - \frac{z}{4}(z^2-1)^{\frac{1}{2}}$$

4.6.52

$$\int z^n \operatorname{arccosh} z\,dz = \frac{z^{n+1}}{n+1}\operatorname{arccosh} z - \frac{1}{n+1}\int \frac{z^{n+1}}{(z^2-1)^{\frac{1}{2}}}dz \qquad (n\neq -1)$$

4.6.53

$$\int z\operatorname{arctanh} z\,dz = \frac{z^2-1}{2}\operatorname{arctanh} z + \frac{z}{2}$$

4.6.54

$$\int z^n \operatorname{arctanh} z\,dz = \frac{z^{n+1}}{n+1}\operatorname{arctanh} z - \frac{1}{n+1}\int \frac{z^{n+1}}{1-z^2}dz \qquad (n\neq -1)$$

4.6.55

$$\int z\operatorname{arccsch} z\,dz = \frac{z^2}{2}\operatorname{arccsh} z \pm \frac{1}{2}(1+z^2)^{\frac{1}{2}} \quad *$$

(according as $\Re z \gtrless 0$)

4.6.56

$$\int z^n \operatorname{arccsch} z\,dz = \frac{z^{n+1}}{n+1}\operatorname{arccsch} z \pm \frac{1}{n+1}\int \frac{z^n}{(z^2+1)^{\frac{1}{2}}}dz \quad * \qquad (n\neq -1)$$

*See page 11.