Lagrange Three Point Interpolation Formula

25.2.11

$$f(x_0+ph)=A_{-1}f_{-1}+A_0f_0+A_1f_1+R_2$$
$$\approx\frac{p(p-1)}{2}f_{-1}+(1-p^2)f_0+\frac{p(p+1)}{2}f_1$$

25.2.12

$$R_2(p)\approx .065h^3f^{(3)}(\xi)\approx .065\Delta^3 \qquad (|p|\le 1)$$

Lagrange Four Point Interpolation Formula

25.2.13

$$f(x_0+ph)=A_{-1}f_{-1}+A_0f_0+A_1f_1+A_2f_2+R_3$$
$$\approx\frac{-p(p-1)(p-2)}{6}f_{-1}+\frac{(p^2-1)(p-2)}{2}f_0$$
$$-\frac{p(p+1)(p-2)}{2}f_1+\frac{p(p^2-1)}{6}f_2$$

25.2.14 $\qquad R_3(p)\approx$

$.024h^4f^{(4)}(\xi)\approx .024\Delta^4 \qquad (0<p<1)$

$.042h^4f^{(4)}(\xi)\approx .042\Delta^4 \qquad (-1<p<0,\ 1<p<2)$

$(x_{-1}<\xi<x_2)$

Lagrange Five Point Interpolation Formula

25.2.15

$$f(x_0+ph)=\sum_{i=-2}^{2}A_if_i+R_4$$
$$\approx\frac{(p^2-1)p(p-2)}{24}f_{-2}-\frac{(p-1)p(p^2-4)}{6}f_{-1}$$
$$+\frac{(p^2-1)(p^2-4)}{4}f_0-\frac{(p+1)p(p^2-4)}{6}f_1$$
$$+\frac{(p^2-1)p(p+2)}{24}f_2$$

25.2.16 $\qquad R_4(p)\approx$

$.012h^5f^{(5)}(\xi)\approx .012\Delta^5 \qquad (|p|<1)$

$.031h^5f^{(5)}(\xi)\approx .031\Delta^5 \qquad (1<|p|<2) \qquad (x_{-2}<\xi<x_2)$

Lagrange Six Point Interpolation Formula

25.2.17

$$f(x_0+ph)=\sum_{i=-2}^{3}A_if_i+R_5$$
$$\approx\frac{-p(p^2-1)(p-2)(p-3)}{120}f_{-2}$$
$$+\frac{p(p-1)(p^2-4)(p-3)}{24}f_{-1}$$
$$-\frac{(p^2-1)(p^2-4)(p-3)}{12}f_0$$
$$+\frac{p(p+1)(p^2-4)(p-3)}{12}f_1-\frac{p(p^2-1)(p+2)(p-3)}{24}f_2$$
$$+\frac{p(p^2-1)(p^2-4)}{120}f_3$$

25.2.18 $\qquad R_5(p)\approx$

$.0049h^6f^{(6)}(\xi)\approx .0049\Delta^6 \qquad (0<p<1)$

$.0071h^6f^{(6)}(\xi)\approx .0071\Delta^6 \qquad (-1<p<0,\ 1<p<2)$

$.024h^6f^{(6)}(\xi)\approx .024\Delta^6 \qquad (-2<p<-1,\ 2<p<3)$

$(x_{-2}<\xi<x_3)$

Lagrange Seven Point Interpolation Formula

25.2.19 $\qquad f(x_0+ph)=\sum_{i=-3}^{3}A_if_i+R_6$

25.2.20

$$R_6(p)\approx\begin{cases}.0025h^7f^{(7)}(\xi)\doteq .0025\Delta^7 & (|p|<1)\\ .0046h^7f^{(7)}(\xi)\approx .0046\Delta^7 & (1<|p|<2)\\ .019h^7f^{(7)}(\xi)\approx .019\Delta^7 & (2<|p|<3)\end{cases}$$

$(x_{-3}<\xi<x_3)$

Lagrange Eight Point Interpolation Formula

25.2.21 $\qquad f(x_0+ph)=\sum_{i=-3}^{4}A_if_i+R_7$

25.2.22

$$R_7(p)\approx\begin{cases}.0011h^8f^{(8)}(\xi)\approx .0011\Delta^8 & (0<p<1)\\ .0014h^8f^{(8)}(\xi)\approx .0014\Delta^8 & (-1<p<0)\ (1<p<2)\\ .0033h^8f^{(8)}(\xi)\approx .0033\Delta^8 & (-2<p<-1)\ (2<p<3)\\ .016h^8f^{(8)}(\xi)\approx .016\Delta^8 & (-3<p<-2)\ (3<p<4)\end{cases}$$

$(x_{-3}<\xi<x_4)$

Aitken's Iteration Method

Let $f(x|x_0, x_1, \ldots, x_k)$ denote the unique polynomial of k^{th} degree which coincides in value with $f(x)$ at $x_0, \ldots, x_k$.

25.2.23

$$f(x|x_0,x_1)=\frac{1}{x_1-x_0}\begin{vmatrix}f_0 & x_0-x\\ f_1 & x_1-x\end{vmatrix}$$

$$f(x|x_0,x_2)=\frac{1}{x_2-x_0}\begin{vmatrix}f_0 & x_0-x\\ f_2 & x_2-x\end{vmatrix}$$

$$f(x|x_0,x_1,x_2)=\frac{1}{x_2-x_1}\begin{vmatrix}f(x|x_0,x_1) & x_1-x\\ f(x|x_0,x_2) & x_2-x\end{vmatrix}$$

$$f(x|x_0,x_1,x_2,x_3)=\frac{1}{x_3-x_2}\begin{vmatrix}f(x|x_0,x_1,x_2) & x_2-x\\ f(x|x_0,x_1,x_3) & x_3-x\end{vmatrix}$$