

4.5.29

$$\cosh \frac{z}{2} = \left(\frac{\cosh z + 1}{2} \right)^{\frac{1}{2}}$$

4.5.30

$$\tanh \frac{z}{2} = \frac{(\cosh z - 1)^{\frac{1}{2}}}{(\cosh z + 1)^{\frac{1}{2}}} = \frac{\cosh z - 1}{\sinh z} = \frac{\sinh z}{\cosh z + 1}$$

Multiple-Angle Formulas

$$4.5.31 \quad \sinh 2z = 2 \sinh z \cosh z = \frac{2 \tanh z}{1 - \tanh^2 z}$$

$$4.5.32 \quad \cosh 2z = 2 \cosh^2 z - 1 = 2 \sinh^2 z + 1 \\ = \cosh^2 z + \sinh^2 z$$

$$4.5.33 \quad \tanh 2z = \frac{2 \tanh z}{1 + \tanh^2 z}$$

$$4.5.34 \quad \sinh 3z = 3 \sinh z + 4 \sinh^3 z$$

$$4.5.35 \quad \cosh 3z = -3 \cosh z + 4 \cosh^3 z$$

$$4.5.36 \quad \sinh 4z = 4 \sinh^3 z \cosh z + 4 \cosh^3 z \sinh z$$

$$4.5.37 \quad \cosh 4z = \cosh^4 z + 6 \sinh^2 z \cosh^2 z + \sinh^4 z$$

Products of Hyperbolic Sines and Cosines

$$4.5.38 \quad 2 \sinh z_1 \sinh z_2 = \cosh (z_1 + z_2) \\ - \cosh (z_1 - z_2)$$

$$4.5.39 \quad 2 \cosh z_1 \cosh z_2 = \cosh (z_1 + z_2) \\ + \cosh (z_1 - z_2)$$

$$4.5.40 \quad 2 \sinh z_1 \cosh z_2 = \sinh (z_1 + z_2) \\ + \sinh (z_1 - z_2)$$

Addition and Subtraction of Two Hyperbolic Functions

4.5.41

$$\sinh z_1 + \sinh z_2 = 2 \sinh \left(\frac{z_1 + z_2}{2} \right) \cosh \left(\frac{z_1 - z_2}{2} \right)$$

4.5.42

$$\sinh z_1 - \sinh z_2 = 2 \cosh \left(\frac{z_1 + z_2}{2} \right) \sinh \left(\frac{z_1 - z_2}{2} \right)$$

4.5.43

$$\cosh z_1 + \cosh z_2 = 2 \cosh \left(\frac{z_1 + z_2}{2} \right) \cosh \left(\frac{z_1 - z_2}{2} \right)$$

4.5.44

$$\cosh z_1 - \cosh z_2 = 2 \sinh \left(\frac{z_1 + z_2}{2} \right) \sinh \left(\frac{z_1 - z_2}{2} \right)$$

4.5.45

$$\tanh z_1 + \tanh z_2 = \frac{\sinh (z_1 + z_2)}{\cosh z_1 \cosh z_2}$$

4.5.46

$$\coth z_1 + \coth z_2 = \frac{\sinh (z_1 + z_2)}{\sinh z_1 \sinh z_2}$$

Relations Between Squares of Hyperbolic Sines and Cosines

4.5.47

$$\sinh^2 z_1 - \sinh^2 z_2 = \sinh (z_1 + z_2) \sinh (z_1 - z_2) \\ = \cosh^2 z_1 - \cosh^2 z_2$$

4.5.48

$$\sinh^2 z_1 + \cosh^2 z_2 = \cosh (z_1 + z_2) \cosh (z_1 - z_2) \\ = \cosh^2 z_1 + \sinh^2 z_2$$

Hyperbolic Functions in Terms of Real and Imaginary Parts

$$(z = x + iy)$$

$$4.5.49 \quad \sinh z = \sinh x \cos y + i \cosh x \sin y$$

$$4.5.50 \quad \cosh z = \cosh x \cos y + i \sinh x \sin y$$

$$4.5.51 \quad \tanh z = \frac{\sinh 2x + i \sin 2y}{\cosh 2x + \cos 2y}$$

$$4.5.52 \quad \coth z = \frac{\sinh 2x - i \sin 2y}{\cosh 2x - \cos 2y}$$

De Moivre's Theorem

$$4.5.53 \quad (\cosh z + \sinh z)^n = \cosh nz + \sinh nz$$

Modulus and Phase (Argument) of Hyperbolic Functions

$$4.5.54 \quad |\sinh z| = (\sinh^2 x + \sin^2 y)^{\frac{1}{2}} \\ = \left[\frac{1}{2} (\cosh 2x - \cos 2y) \right]^{\frac{1}{2}}$$

$$4.5.55 \quad \arg \sinh z = \arctan (\coth x \tan y)$$

$$4.5.56 \quad |\cosh z| = (\sinh^2 x + \cos^2 y)^{\frac{1}{2}} \\ = \left[\frac{1}{2} (\cosh 2x + \cos 2y) \right]^{\frac{1}{2}}$$

$$4.5.57 \quad \arg \cosh z = \arctan (\tanh x \tan y)$$

$$4.5.58 \quad |\tanh z| = \left(\frac{\cosh 2x - \cos 2y}{\cosh 2x + \cos 2y} \right)^{\frac{1}{2}}$$

$$4.5.59 \quad \arg \tanh z = \arctan \left(\frac{\sin 2y}{\sinh 2x} \right)$$