Approximations in Terms of Chebyshev Polynomials [12]

4.4.50 $-1\leq x\leq 1$

$T^*_n(x)=\cos n\theta, \quad \cos\theta=2x-1$ (see chapter 22)

$$\arctan x = x\sum_{n=0}^{\infty} A_n T^*_n(x^2)$$

n	A_n	n	A_n
0	.88137 3587	6	.00000 3821
1	−.10589 2925	7	−.00000 0570
2	.01113 5843	8	.00000 0086
3	−.00138 1195	9	−.00000 0013
4	.00018 5743	10	.00000 0002
5	−.00002 6215		

*For $x>1$, use $\arctan x=\frac{1}{2}\pi-\arctan(1/x)$

4.4.51 $-\frac{1}{2}\sqrt{2}\leq x\leq\frac{1}{2}\sqrt{2}$

$$\arcsin x = x\sum_{n=0}^{\infty} A_n T^*_n(2x^2)$$

$$0\leq x\leq\tfrac{1}{2}\sqrt{2}$$

$$\arccos x=\tfrac{1}{2}\pi - x\sum_{n=0}^{\infty} A_n T^*_n(2x^2)$$

n	A_n	n	A_n
0	1.05123 1959	5	.00000 5881
1	.05494 6487	6	.00000 0777
2	.00408 0631	7	.00000 0107
3	.00040 7890	8	.00000 0015
4	.00004 6985	9	.00000 0002

For $\frac{1}{2}\sqrt{2}\leq x\leq 1$, use $\arcsin x=\arccos(1-x^2)^{\frac{1}{2}}$, $\arccos x=\arcsin(1-x^2)^{\frac{1}{2}}$.

Differentiation Formulas

4.4.52 $$\frac{d}{dz}\arcsin z=(1-z^2)^{-\frac{1}{2}}$$

4.4.53 $$\frac{d}{dz}\arccos z=-(1-z^2)^{-\frac{1}{2}}$$

4.4.54 $$\frac{d}{dz}\arctan z=\frac{1}{1+z^2}$$

4.4.55 $$\frac{d}{dz}\operatorname{arccot} z=\frac{-1}{1+z^2}$$

4.4.56 $$\frac{d}{dz}\operatorname{arcsec} z=\frac{1}{z(z^2-1)^{\frac{1}{2}}}$$

4.4.57 $$\frac{d}{dz}\operatorname{arccsc} z=-\frac{1}{z(z^2-1)^{\frac{1}{2}}}$$

Integration Formulas

4.4.58 $$\int\arcsin z\,dz=z\arcsin z+(1-z^2)^{\frac{1}{2}}$$

4.4.59 $$\int\arccos z\,dz=z\arccos z-(1-z^2)^{\frac{1}{2}}$$

4.4.60 $$\int\arctan z\,dz=z\arctan z-\tfrac{1}{2}\ln(1+z^2)$$

4.4.61
$$\int\operatorname{arccsc} z\,dz=z\operatorname{arccsc} z\pm\ln[z+(z^2-1)^{\frac{1}{2}}]$$
$$\left[\begin{array}{c}0<\operatorname{arccsc} z<\frac{\pi}{2}\\ -\frac{\pi}{2}<\operatorname{arccsc} z<0\end{array}\right]$$

4.4.62
$$\int\operatorname{arcsec} z\,dz=z\operatorname{arcsec} z\mp\ln[z+(z^2-1)^{\frac{1}{2}}]$$
$$\left[\begin{array}{c}0<\operatorname{arcsec} z<\frac{\pi}{2}\\ \frac{\pi}{2}<\operatorname{arcsec} z<\pi\end{array}\right]$$

4.4.63
$$\int\operatorname{arccot} z\,dz=z\operatorname{arccot} z+\tfrac{1}{2}\ln(1+z^2)$$

4.4.64
$$\int z\arcsin z\,dz=\left(\frac{z^2}{2}-\frac{1}{4}\right)\arcsin z+\frac{z}{4}(1-z^2)^{\frac{1}{2}}$$

4.4.65
$$\int z^n\arcsin z\,dz=\frac{z^{n+1}}{n+1}\arcsin z-\frac{1}{n+1}\int\frac{z^{n+1}}{(1-z^2)^{\frac{1}{2}}}dz \qquad (n\neq-1)$$

4.4.66
$$\int z\arccos z\,dz=\left(\frac{z^2}{2}-\frac{1}{4}\right)\arccos z-\frac{z}{4}(1-z^2)^{\frac{1}{2}}$$

4.4.67
$$\int z^n\arccos z\,dz=\frac{z^{n+1}}{n+1}\arccos z+\frac{1}{n+1}\int\frac{z^{n+1}}{(1-z^2)^{\frac{1}{2}}}dz \qquad (n\neq-1)$$

4.4.68
$$\int z\arctan z\,dz=\frac{1}{2}(1+z^2)\arctan z-\frac{z}{2}$$

[12] The approximations **4.4.50** to **4.4.51** are from C. W. Clenshaw, Polynomial approximations to elementary functions, Math. Tables Aids Comp. 8, 143–147 (1954) (with permission).

*See page II.