Fundamental Property

The general solutions of the equations

$$\sin t = z$$

$$\cos t = z$$

$$\tan t = z$$

are respectively

4.4.10 $t = \text{Arcsin } z = (-1)^k \arcsin z + k\pi$

4.4.11 $t = \text{Arccos } z = \pm \arccos z + 2k\pi$

4.4.12
$t = \text{Arctan } z = \arctan z + k\pi \qquad (z^2 \neq -1)$

where k is an arbitrary integer.

4.4.13 Interval containing principal value

y	x positive or zero	x negative
arcsin x and arctan x	$0 \leq y \leq \pi/2$	$-\pi/2 \leq y < 0$
*arccos x and arcsec x	$0 \leq y \leq \pi/2$	$\pi/2 < y \leq \pi$
*arccot x and arccsc x	$0 \leq y \leq \pi/2$	$-\pi/2 \leq y < 0$

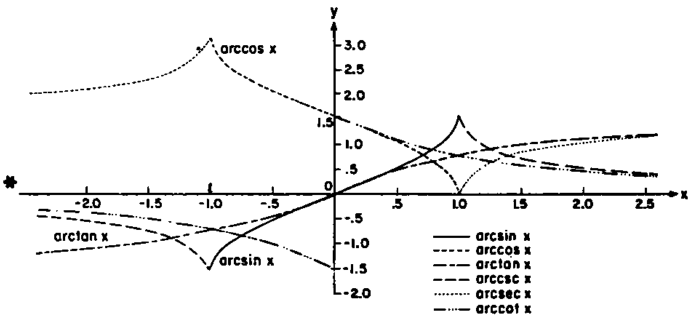

FIGURE 4.5. *Inverse circular functions.*

Functions of Negative Arguments

4.4.14 $\arcsin(-z) = -\arcsin z$

4.4.15 $\arccos(-z) = \pi - \arccos z$

4.4.16 $\arctan(-z) = -\arctan z$

4.4.17 $\text{arccsc}(-z) = -\text{arccsc } z$

4.4.18 $\text{arcsec}(-z) = \pi - \text{arcsec } z$

*4.4.19 $\text{arccot}(-z) = -\text{arccot } z$

Relation to Inverse Hyperbolic Functions (see 4.6.14 to 4.6.19)

4.4.20 $\text{Arcsin } z = -i \text{ Arcsinh } iz$

4.4.21 $\text{Arccos } z = \pm i \text{ Arccosh } z$

4.4.22 $\text{Arctan } z = -i \text{ Arctanh } iz \qquad (z^2 \neq -1)$

4.4.23 $\text{Arccsc } z = i \text{ Arccsch } iz$

4.4.24 $\text{Arcsec } z = \pm i \text{ Arcsech } z$

4.4.25 $\text{Arccot } z = i \text{ Arccoth } iz$

Logarithmic Representations

4.4.26 $\text{Arcsin } x = -i \text{ Ln}[(1-x^2)^{\frac{1}{2}} + ix] \qquad (x^2 \leq 1)$

4.4.27 $\text{Arccos } x = -i \text{ Ln}[x + i(1-x^2)^{\frac{1}{2}}] \qquad (x^2 \leq 1)$

4.4.28 $\text{Arctan } x = \frac{i}{2} \text{ Ln } \frac{1-ix}{1+ix} = \frac{i}{2} \text{ Ln } \frac{i+x}{i-x}$

$(x \text{ real})$

4.4.29 $\text{Arccsc } x = -i \text{ Ln}\left[\frac{(x^2-1)^{\frac{1}{2}}+i}{x}\right] \qquad (x^2 \geq 1)$

4.4.30 $\text{Arcsec } x = -i \text{ Ln}\left[\frac{1+i(x^2-1)^{\frac{1}{2}}}{x}\right] \qquad (x^2 \geq 1)$

4.4.31 $\text{Arccot } x = \frac{i}{2} \text{Ln}\left(\frac{ix+1}{ix-1}\right) = \frac{i}{2} \text{Ln}\left(\frac{x-i}{x+i}\right)$

$(x \text{ real})$

Addition and Subtraction of Two Inverse Circular Functions

4.4.32

$$\text{Arcsin } z_1 \pm \text{Arcsin } z_2 = \text{Arcsin}[z_1(1-z_2^2)^{\frac{1}{2}} \pm z_2(1-z_1^2)^{\frac{1}{2}}]$$

4.4.33

$$\text{Arccos } z_1 \pm \text{Arccos } z_2 = \text{Arccos}\{z_1 z_2 \mp [(1-z_1^2)(1-z_2^2)]^{\frac{1}{2}}\}$$

4.4.34

$$\text{Arctan } z_1 \pm \text{Arctan } z_2 = \text{Arctan}\left(\frac{z_1 \pm z_2}{1 \mp z_1 z_2}\right)$$

4.4.35

$$\text{Arcsin } z_1 \pm \text{Arccos } z_2 = \text{Arcsin}\{z_1 z_2 \pm [(1-z_1^2)(1-z_2^2)]^{\frac{1}{2}}\}$$
$$= \text{Arccos}[z_2(1-z_1^2)^{\frac{1}{2}} \mp z_1(1-z_2^2)^{\frac{1}{2}}]$$

4.4.36

$$\text{Arctan } z_1 \pm \text{Arccot } z_2 = \text{Arctan}\left(\frac{z_1 z_2 \pm 1}{z_2 \mp z_1}\right) = \text{Arccot}\left(\frac{z_2 \mp z_1}{z_1 z_2 \pm 1}\right)$$

Inverse Circular Functions in Terms of Real and Imaginary Parts

4.4.37

$$\text{Arcsin } z = k\pi + (-1)^k \arcsin \beta + (-1)^k i \ln[\alpha + (\alpha^2-1)^{\frac{1}{2}}]$$

4.4.38

$$\text{Arccos } z = 2k\pi \pm \{\arccos \beta - i \ln[\alpha + (\alpha^2-1)^{\frac{1}{2}}]\}$$

*See page II.