Table 22.2 **Coefficients for the Ultraspherical Polynomials $C_n^{(\alpha)}(x)$ and for x^n in terms of $C_m^{(\alpha)}(x)$**

$$C_n^{(\alpha)}(x)=a_n^{-1}\sum_{m=0}^{n} c_m x^m \text{ and } x^n=b_n^{-1}\sum_{m=0}^{n} d_m C_m^{(\alpha)}(x) \qquad (\alpha\neq 0)$$

		x^0	x^1	x^2	x^3	x^4	x^5	x^6	
	b_n	1	2α	$2(\alpha)_2$	$4(\alpha)_3$	$4(\alpha)_4$	$8(\alpha)_5$	$8(\alpha)_6$	
$C_0^{(\alpha)}$	a_n 1	1 1		α		$3\alpha(\alpha+3)$		$15\alpha(\alpha+4)(\alpha+5)$	$C_0^{(\alpha)}$
$C_1^{(\alpha)}$	1		2α 1		$3(\alpha+1)$		$15(\alpha+1)(\alpha+4)$		$C_1^{(\alpha)}$
$C_2^{(\alpha)}$	1	$-\alpha$		$2(\alpha)_2$ 1		$6(\alpha+2)$		$45(\alpha+2)(\alpha+5)$	$C_2^{(\alpha)}$
$C_3^{(\alpha)}$	3		$-6(\alpha)_2$		$4(\alpha)_3$ 3		$30(\alpha+3)$		$C_3^{(\alpha)}$
$C_4^{(\alpha)}$	6	$3(\alpha)_2$		$-12(\alpha)_3$		$4(\alpha)_4$ 6		$90(\alpha+4)$	$C_4^{(\alpha)}$
$C_5^{(\alpha)}$	15		$15(\alpha)_3$		$-20(\alpha)_4$		$4(\alpha)_5$ 30		$C_5^{(\alpha)}$
$C_6^{(\alpha)}$	90	$-15(\alpha)_3$		$90(\alpha)_4$		$-60(\alpha)_5$		$8(\alpha)_6$ 90	$C_6^{(\alpha)}$
		x^0	x^1	x^2	x^3	x^4	x^5	x^6	

$$(\alpha)_n=\alpha(\alpha+1)(\alpha+2)\ .\ .\ .\ (\alpha+n-1)$$

$$C_3^{(2)}(x)=\frac{1}{3}\,[4(2)_3x^3-6(2)_2x] \qquad x^3=\frac{1}{4(2)_3}\,[3(3)C_1^{(2)}(x)+3C_3^{(2)}(x)]$$

$$C_3^{(2)}(x)=\frac{1}{3}\,[96x^3-36x] \qquad x^3=\frac{1}{96}\,[9C_1^{(2)}(x)+3C_3^{(2)}(x)]$$