

$$4.3.131 \quad \int \frac{dz}{a+b \sin z} = \frac{2}{(a^2-b^2)^{\frac{1}{2}}} \arctan \frac{a \tan \left(\frac{z}{2}\right) + b}{(a^2-b^2)^{\frac{1}{2}}} \quad (a^2 > b^2)$$

$$= \frac{1}{(b^2-a^2)^{\frac{1}{2}}} \ln \left[ \frac{a \tan \left(\frac{z}{2}\right) + b - (b^2-a^2)^{\frac{1}{2}}}{a \tan \left(\frac{z}{2}\right) + b + (b^2-a^2)^{\frac{1}{2}}} \right] \quad (b^2 > a^2)$$

$$4.3.132 \quad \int \frac{dz}{1 \pm \sin z} = \mp \tan \left( \frac{\pi}{4} \mp \frac{z}{2} \right)$$

$$4.3.133 \quad \int \frac{dz}{a+b \cos z} = \frac{2}{(a^2-b^2)^{\frac{1}{2}}} \arctan \frac{(a-b) \tan \frac{z}{2}}{(a^2-b^2)^{\frac{1}{2}}} \quad (a^2 > b^2)$$

$$= \frac{1}{(b^2-a^2)^{\frac{1}{2}}} \ln \left[ \frac{(b-a) \tan \frac{z}{2} + (b^2-a^2)^{\frac{1}{2}}}{(b-a) \tan \frac{z}{2} - (b^2-a^2)^{\frac{1}{2}}} \right] \quad (b^2 > a^2)$$

$$4.3.134 \quad \int \frac{dz}{1+\cos z} = \tan \frac{z}{2}$$

$$4.3.135 \quad \int \frac{dz}{1-\cos z} = -\cot \frac{z}{2}$$

$$4.3.136 \quad \int e^{az} \sin bz \, dz = \frac{e^{az}}{a^2+b^2} (a \sin bz - b \cos bz)$$

$$4.3.137 \quad \int e^{az} \cos bz \, dz = \frac{e^{az}}{a^2+b^2} (a \cos bz + b \sin bz)$$

$$4.3.138 \quad \int e^{az} \sin^n bz \, dz = \frac{e^{az} \sin^{n-1} bz}{a^2+n^2b^2} (a \sin bz - nb \cos bz) \\ + \frac{n(n-1)b^2}{a^2+n^2b^2} \int e^{az} \sin^{n-2} bz \, dz$$

$$4.3.139 \quad \int e^{az} \cos^n bz \, dz = \frac{e^{az} \cos^{n-1} bz}{a^2+n^2b^2} (a \cos bz + nb \sin bz) \\ + \frac{n(n-1)b^2}{a^2+n^2b^2} \int e^{az} \cos^{n-2} bz \, dz$$

#### Definite Integrals

$$4.3.140 \quad \int_0^\pi \sin mt \sin nt \, dt = 0 \\ (m \neq n, \quad m \text{ and } n \text{ integers})$$

$$\int_0^\pi \cos mt \cos nt \, dt = 0$$

$$4.3.141 \quad \int_0^\pi \sin^2 nt \, dt = \int_0^\pi \cos^2 nt \, dt = \frac{\pi}{2} \\ (n \text{ an integer, } n \neq 0)$$

$$4.3.142 \quad \int_0^\infty \frac{\sin mt}{t} \, dt = \frac{\pi}{2} \quad (m > 0) \\ = 0 \quad (m = 0) \\ = -\frac{\pi}{2} \quad (m < 0)$$

$$4.3.143 \quad \int_0^\infty \frac{\cos at - \cos bt}{t} \, dt = \ln(b/a)$$

$$4.3.144 \quad \int_0^\infty \sin t^2 \, dt = \int_0^\infty \cos t^2 \, dt = \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

$$4.3.145 \quad \int_0^{\pi/2} \ln \sin t \, dt = \int_0^{\pi/2} \ln \cos t \, dt = -\frac{\pi}{2} \ln 2$$

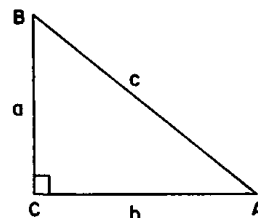
$$4.3.146 \quad \int_0^\infty \frac{\cos mt}{1+t^2} \, dt = \frac{\pi}{2} e^{-m}$$

(See chapters 5 and 7 for other integrals involving circular functions.)

(See [5.3] for Fourier transforms.)

#### 4.3.147

#### Formulas for Solution of Plane Right Triangles



If  $A$ ,  $B$  and  $C$  are the vertices ( $C$  the right angle), and  $a$ ,  $b$  and  $c$  the sides opposite respectively,

$$\sin A = \frac{a}{c} = \frac{1}{\csc A}$$

$$\cos A = \frac{b}{c} = \frac{1}{\sec A}$$

$$\tan A = \frac{a}{b} = \frac{1}{\cot A}$$

$$\text{versine } A = \text{vers } A = 1 - \cos A$$

$$\text{coversine } A = \text{covers } A = 1 - \sin A$$

$$\text{haversine } A = \text{hav } A = \frac{1}{2} \text{vers } A$$

$$\text{exsecant } A = \text{exsec } A = \sec A - 1$$