



FIGURE 22.7. Chebyshev Polynomials $U_n(x)$, $n=1(1)5$.

22.5.33 $T_n(x) = \frac{n}{2} C_n^{(0)}(x)$

22.5.34 $U_n(x) = C_n^{(1)}(x)$

Legendre Polynomials

22.5.35 $P_n(x) = P_n^{(0,0)}(x)$

22.5.36 $P_n(x) = C_n^{(1/2)}(x)$

22.5.37

$$\frac{d^m}{dx^m} [P_n(x)] = 1 \cdot 3 \dots (2m-1) C_{n-m}^{(m+\frac{1}{2})}(x) \quad (m \leq n)$$

Generalized Laguerre Polynomials

22.5.38 $L_n^{(-1/2)}(x) = \frac{(-1)^n}{n! 2^{2n}} H_{2n}(\sqrt{x})$

22.5.39 $L_n^{(1/2)}(x) = \frac{(-1)^n}{n! 2^{2n+1} \sqrt{x}} H_{2n+1}(\sqrt{x})$

Hermite Polynomials

22.5.40 $H_{2m}(x) = (-1)^m 2^{2m} m! L_m^{(-1/2)}(x^2)$

22.5.41 $H_{2m+1}(x) = (-1)^m 2^{2m+1} m! x L_m^{(1/2)}(x^2)$

Orthogonal Polynomials as Hypergeometric Functions (see chapter 15)

$$f_n(x) = dF(a, b; c; g(x))$$

For each of the listed polynomials there are numerous other representations in terms of hypergeometric functions.

	$f_n(x)$	d	a	b	c	$g(x)$
22.5.42	$P_n^{(\alpha, \beta)}(x)$	$\binom{n+\alpha}{n}$	$-n$	$n+\alpha+\beta+1$	$\alpha+1$	$\frac{1-x}{2}$
22.5.43	$P_n^{(\alpha, \beta)}(x)$	$\binom{2n+\alpha+\beta}{n} \left(\frac{x-1}{2}\right)^n$	$-n$	$-n-\alpha$	$-2n-\alpha-\beta$	$\frac{2}{1-x}$
22.5.44	$P_n^{(\alpha, \beta)}(x)$	$\binom{n+\alpha}{n} \left(\frac{1+x}{2}\right)^n$	$-n$	$-n-\beta$	$\alpha+1$	$\frac{x-1}{x+1}$
22.5.45	$P_n^{(\alpha, \beta)}(x)$	$\binom{n+\beta}{n} \left(\frac{x-1}{2}\right)^n$	$-n$	$-n-\alpha$	$\beta+1$	$\frac{x+1}{x-1}$
22.5.46	$C_n^{(\alpha)}(x)$	$\frac{\Gamma(n+2\alpha)}{n! \Gamma(2\alpha)}$	$-n$	$n+2\alpha$	$\alpha+\frac{1}{2}$	$\frac{1-x}{2}$
22.5.47	$T_n(x)$	1	$-n$	n	$\frac{1}{2}$	$\frac{1-x}{2}$
22.5.48	$U_n(x)$	$n+1$	$-n$	$n+2$	*	$\frac{1-x}{2}$
22.5.49	$P_n(x)$	1	$-n$	$n+1$	1	$\frac{1-x}{2}$
22.5.50	$P_n(x)$	$\binom{2n}{n} \left(\frac{x-1}{2}\right)^n$	$-n$	$-n$	$-2n$	$\frac{2}{1-x}$
22.5.51	$P_n(x)$	$\binom{2n}{n} \left(\frac{x}{2}\right)^n$	$-\frac{n}{2}$	$\frac{1-n}{2}$	$\frac{1}{2}-n$	$\frac{1}{x^2}$
22.5.52	$P_{2n}(x)$	$(-1)^n \frac{(2n)!}{2^{2n} (n!)^2}$	$-n$	$n+\frac{1}{2}$	$\frac{1}{2}$	x^2
22.5.53	$P_{2n+1}(x)$	$(-1)^n \frac{(2n+1)!}{2^{2n} (n!)^2} x$	$-n$	$n+\frac{3}{2}$	$\frac{3}{2}$	x^2

*See page II.