

Table 20.1 CHARACTERISTIC VALUES, JOINING FACTORS, SOME CRITICAL VALUES  
EVEN SOLUTIONS

$r$	$q$	$a_r$	$ce_r(0, q)$	$ce_r(\frac{1}{2}\pi, q)$	$(4q)^{\frac{1}{2}r}g_{e,r}(q)$	$(4q)^rf_{e,r}(q)$
0	0	0.00000 000	(-1) 7.07106 781	(-1) 7.07106 78	(-1) 7.97884 56	$\infty$
	5	5.80004 602	(-2) 4.48001 817	1.33484 87	1.97009 00	(-3) 1.86132 97
	10	- 13.93697 996	(-3) 7.62651 757	1.46866 05	2.40237 95	(-5) 5.54257 96
	15	- 22.51303 776	(-3) 1.93250 832	1.55010 82	2.68433 53	(-6) 3.59660 89
	20	- 31.31339 007	(-4) 6.03743 829	1.60989 09	2.90011 25	(-7) 3.53093 01
	25	- 40.25677 955	(-4) 2.15863 018	1.65751 03	3.07743 91	(-8) 4.53098 68
2	0	4.00000 000	1.00000 000	-1.00000 00	( 1) 1.27661 53	( 1) 8.14873 31
	5	7.44910 974	(-1) 7.35294 308	(-1) -7.24488 15	( 1) 2.63509 89	( 2) 1.68665 79
	10	7.71736 985	(-1) 2.45888 349	(-1) -9.26759 26	( 1) 7.22275 58	( 1) 6.89192 56
	15	5.07798 320	(-2) 7.87928 278	-1.01996 62	( 2) 1.32067 71	( 1) 1.73770 48
	20	+ 1.15428 288	(-2) 2.86489 431	-1.07529 32	( 2) 1.98201 14	4.29953 32
	25	- 3.52216 473	(-2) 1.15128 663	-1.11627 90	( 2) 2.69191 26	1.11858 69
10	0	100.00000 000	1.00000 000	-1.00000 00	(12) 1.51800 43	( 23) 2.30433 72
	5	100.12636 922	1.02599 503	(-1) -9.75347 49	(12) 1.48332 54	( 23) 2.31909 77
	10	100.50677 002	1.05381 599	(-1) -9.51645 32	(12) 1.45530 39	( 23) 2.36418 54
	15	101.14520 345	1.08410 631	(-1) -9.28548 06	(12) 1.43299 34	( 23) 2.44213 04
	20	102.04891 602	1.11778 863	(-1) -9.05710 78	(12) 1.41537 24	( 23) 2.55760 55
	25	103.23020 480	1.15623 992	(-1) -8.82691 92	(12) 1.40118 52	( 23) 2.71854 15

$r$	$q$	$a_r$	$ce_r(0, q)$	$ce'_r(\frac{1}{2}\pi, q)$	$(4q)^{\frac{1}{2}r}g_{e,r}(q)$	$(4q)^rf_{e,r}(q)$
1	0	1.00000 000	1.00000 000	-1.00000 00	1.59576 91	2.54647 91
	5	+ 1.85818 754	(-1) 2.56542 879	-3.46904 21	7.26039 84	1.02263 46
	10	- 2.39914 240	(-2) 5.35987 478	-4.85043 83	( 1) 1.35943 49	(-2) 9.72660 12
	15	- 8.10110 513	(-2) 1.50400 665	-5.76420 64	( 1) 1.91348 51	(-2) 1.19739 95
	20	- 14.49130 142	(-3) 5.05181 376	-6.49056 58	( 1) 2.42144 01	(-3) 1.84066 20
	25	- 21.31489 969	(-3) 1.91105 151	-7.10674 15	( 1) 2.89856 94	(-4) 3.33747 55
5	0	25.00000 000	1.00000 000	-5.00000 00	( 4) 4.90220 27	( 8) 4.80631 83
	5	25.54997 175	1.12480 725	-5.39248 61	( 4) 4.43075 22	( 8) 5.11270 71
	10	27.70376 873	1.25801 994	-5.32127 65	( 4) 4.19827 66	( 8) 6.83327 77
	15	31.95782 125	1.19343 223	-5.11914 99	( 4) 5.25017 04	( 9) 1.18373 72
	20	36.64498 973	(-1) 9.36575 531	-5.77867 52	( 4) 8.96243 97	( 9) 1.85341 57
	25	40.05019 099	(-1) 6.10694 310	-7.05988 45	( 5) 1.71582 55	( 9) 2.09679 12
15	0	225.00000 000	1.00000 000	( 1) 1.50000 00	(20) 5.60156 72	( 40) 2.09183 70
	5	225.05581 248	1.01129 373	( 1) 1.51636 57	(20) 5.54349 84	( 40) 2.09575 00
	10	225.22335 698	1.02287 828	( 1) 1.53198 84	(20) 5.49405 67	( 40) 2.10754 45
	15	225.50295 624	1.03479 365	( 1) 1.54687 43	(20) 5.45287 72	( 40) 2.12738 84
	20	225.89515 341	1.04708 434	( 1) 1.56102 79	(20) 5.41964 26	( 40) 2.15556 69
	25	226.40072 004	1.05980 044	( 1) 1.57444 72	(20) 5.39407 68	( 40) 2.19249 18

Compiled from National Bureau of Standards, Tables relating to Mathieu functions, Columbia Univ. Press, New York, N.Y., 1951 (with permission).

$q^{-\frac{1}{2}}\sqrt{r}$	$a_r + 2q - (4r+2)\sqrt{q}$							$\langle q \rangle$
	0	1	2	5	10	15		
0.16	-0.25532 994	-1.30027 212	-3.45639 483	-17.84809 551	-76.04295 314	- 80.93485 048	39	
0.12	-0.25393 098	-1.28658 972	-3.39777 782	-16.92019 225	-76.84607 855	-141.64507 841	69	
0.08	-0.25257 851	-1.27371 191	-3.34441 938	-16.25305 645	-63.58155 264	-162.30500 052	156	
0.04	-0.25126 918	-1.26154 161	-3.29538 745	-15.70968 373	-58.63500 546	-132.08298 271	625	
0.00	-0.25000 000	-1.25000 000	-3.25000 000	-15.25000 000	-55.25000 000	-120.25000 000	$\infty$	

For  $g_{e,r}$  and  $f_{e,r}$  see 20.8.12.

$\langle q \rangle$  = nearest integer to  $q$ .

Compiled from G. Blanch and I. Rhodes, Table of characteristic values of Mathieu's equation for large values of the parameter, Jour. Wash. Acad. Sci., 45, 6, 1955 (with permission).