$$\frac{ce_1(0,q)}{ce_1'(\frac{1}{2}\pi,q)}\sim -4\sqrt{2}e^{-2\sqrt{q}}\left(1+\frac{3}{16\sqrt{q}}+\frac{45}{256q}+\ldots\right)$$

$$\frac{ce_3(0,q)}{ce_3'(\frac{1}{2}\pi,q)}\sim \frac{64}{3}\,q\sqrt{2}\,e^{-2\sqrt{q}}\left(1-\frac{3}{16\sqrt{q}}+\frac{47}{128q}+\ldots\right)$$

20.9.24

$$\frac{se_1'(0,q)}{se_1(\frac{1}{2}\pi,q)}\sim 4\,q\sqrt{2}\,e^{-2\sqrt{q}}\left(1-\frac{3}{16\sqrt{q}}-\frac{11}{256q}+\ldots\right)$$

$$\frac{se_3'(0,q)}{se_3(\frac{1}{2}\pi,q)}\sim -64\,q\sqrt{2}\,e^{-2\sqrt{q}}\left(1-\frac{21}{16\sqrt{q}}-\frac{17}{128q}+\ldots\right)$$

$$\frac{se_2'(0,q)}{se_2'(\frac{1}{2}\pi,q)}\sim -8\,q\sqrt{2}\,e^{-2\sqrt{q}}\left(1-\frac{9}{16\sqrt{q}}-\frac{39}{256q}+\ldots\right)$$

$$\frac{se_4'(0,q)}{se_4'(\frac{1}{2}\pi,q)}\sim \frac{128}{3}\,q\sqrt{2}e^{-2\sqrt{q}}\left(1-\frac{31}{16\sqrt{q}}-\frac{15}{128q}+\ldots\right)$$

For higher orders, these ratios are increasingly more difficult to obtain. One method of estimating values at the origin is to evaluate both **20.9.11** and **20.9.15** for some x where both expansions are satisfactory, and so to use **20.9.11** as a means to solve for $ce_r(0, q)$; similarly for $se_r'(0, q)$.

Other asymptotic expansions, valid over various regions of the complex z-plane, for real values of a, q, have been given by Langer [20.25]. It is not always easy, however, to determine the linear combinations of Langer's solutions which coincide with those defined here.