4.3.46 Circular Functions for Certain Angles

	0 0°	π/12 15°	π/6 30°	π/4 45°	π/3 60°
sin	0	$\frac{\sqrt{2}}{4}(\sqrt{3}-1)$	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	1	$\frac{\sqrt{2}}{4}(\sqrt{3}+1)$	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2
tan	0	$2-\sqrt{3}$	$\sqrt{3}/3$	1	$\sqrt{3}$
csc	∞	$\sqrt{2}(\sqrt{3}+1)$	2	$\sqrt{2}$	$2\sqrt{3}/3$
sec	1	$\sqrt{2}(\sqrt{3}-1)$	$2\sqrt{3}/3$	$\sqrt{2}$	2
cot	∞	$2+\sqrt{3}$	$\sqrt{3}$	1	$\sqrt{3}/3$

	5π/12 75°	π/2 90°	7π/12 105°	2π/3 120°
sin	$\frac{\sqrt{2}}{4}(\sqrt{3}+1)$	1	$\frac{\sqrt{2}}{4}(\sqrt{3}+1)$	$\sqrt{3}/2$
cos	$\frac{\sqrt{2}}{4}(\sqrt{3}-1)$	0	$\frac{-\sqrt{2}}{4}(\sqrt{3}-1)$	$-1/2$
tan	$2+\sqrt{3}$	∞	$-(2+\sqrt{3})$	$-\sqrt{3}$
csc	$\sqrt{2}(\sqrt{3}-1)$	1	$\sqrt{2}(\sqrt{3}-1)$	$2\sqrt{3}/3$
sec	$\sqrt{2}(\sqrt{3}+1)$	∞	$-\sqrt{2}(\sqrt{3}+1)$	-2
cot	$2-\sqrt{3}$	0	$-(2-\sqrt{3})$	$-\sqrt{3}/3$

	3π/4 135°	5π/6 150°	11π/12 165°	π 180°
sin	$\sqrt{2}/2$	1/2	$\frac{\sqrt{2}}{4}(\sqrt{3}-1)$	0
cos	$-\sqrt{2}/2$	$-\sqrt{3}/2$	$\frac{-\sqrt{2}}{4}(\sqrt{3}+1)$	-1
tan	-1	$-\sqrt{3}/3$	$-(2-\sqrt{3})$	0
csc	$\sqrt{2}$	2	$\sqrt{2}(\sqrt{3}+1)$	∞
sec	$-\sqrt{2}$	$-2\sqrt{3}/3$	$-\sqrt{2}(\sqrt{3}-1)$	-1
cot	-1	$-\sqrt{3}$	$-(2+\sqrt{3})$	∞

Euler's Formula

4.3.47 $e^z=e^{x+iy}=e^x(\cos y+i\sin y)$

De Moivre's Theorem

4.3.48 $(\cos z+i\sin z)^\nu=\cos \nu z+i\sin \nu z$

$(-\pi<\mathscr{R}z\leq\pi$ unless ν is an integer$)$

Relation to Hyperbolic Functions (see 4.5.7 to 4.5.12)

4.3.49 $\sin z=-i\sinh iz$

4.3.50 $\cos z=\cosh iz$

4.3.51 $\tan z=-i\tanh iz$

4.3.52 $\csc z=i\,\mathrm{csch}\,iz$

4.3.53 $\sec z=\mathrm{sech}\,iz$

4.3.54 $\cot z=i\coth iz$

Circular Functions in Terms of Real and Imaginary Parts

4.3.55 $\sin z=\sin x\cosh y+i\cos x\sinh y$

4.3.56 $\cos z=\cos x\cosh y-i\sin x\sinh y$

4.3.57 $\tan z=\frac{\sin 2x+i\sinh 2y}{\cos 2x+\cosh 2y}$

4.3.58 $\cot z=\frac{\sin 2x-i\sinh 2y}{\cosh 2y-\cos 2x}$

Modulus and Phase (Argument) of Circular Functions

4.3.59 $|\sin z|=(\sin^2 x+\sinh^2 y)^{\frac{1}{2}}$

$=[\tfrac{1}{2}(\cosh 2y-\cos 2x)]^{\frac{1}{2}}$

4.3.60 $\arg\sin z=\arctan(\cot x\tanh y)$

4.3.61 $|\cos z|=(\cos^2 x+\sinh^2 y)^{\frac{1}{2}}$

$=[\tfrac{1}{2}(\cosh 2y+\cos 2x)]^{\frac{1}{2}}$

4.3.62 $\arg\cos z=-\arctan(\tan x\tanh y)$

4.3.63 $|\tan z|=\left(\frac{\cosh 2y-\cos 2x}{\cosh 2y+\cos 2x}\right)^{\frac{1}{2}}$

4.3.64 $\arg\tan z=\arctan\left(\frac{\sinh 2y}{\sin 2x}\right)$

Series Expansions

4.3.65

$$\sin z=z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}+\ldots \qquad (|z|<\infty)$$

4.3.66

$$\cos z=1-\frac{z^2}{2!}+\frac{z^4}{4!}-\frac{z^6}{6!}+\ldots \qquad (|z|<\infty)$$