20.7.21 $$ce_r(z, q)=\sigma_r\int_0^{\pi/2}\cosh\,(2\sqrt{q}\sin z\sin t)[(1-p)+p\cos z\cos t]ce_r(t, q)dt$$

20.7.22 $$se_r(z, q)=\rho_r\int_0^{\pi/2}\sin\left(2\sqrt{q}\cos z\cos t+p\frac{\pi}{2}\right)\sin z\sin t\; se_r(t, q)dt$$

20.7.23 $$se_r(z, q)=\sigma_r\int_0^{\pi/2}\sinh\,(2\sqrt{q}\sin z\sin t)[(1-p)\cos z\cos t+p]se_r(t, q)dt$$

where

20.7.24 $\rho_r=\frac{2}{\pi}ce_{2s}\left(\frac{\pi}{2}, q\right)/A_0^{2s}(q)$; $p=0$ $\rho_r=\frac{-2}{\pi}ce'_{2s+1}\left(\frac{\pi}{2}, q\right)/\sqrt{q}A_1^{2s+1}(q)$ if $p=1$, for functions $ce_r(z, q)$

$\rho_r=\frac{-4}{\pi}se'_{2s}\left(\frac{\pi}{2}, q\right)/\sqrt{q}B_2^{2s}(q)$; $\rho_r=\frac{4}{\pi}se_{2s+1}\left(\frac{\pi}{2}, q\right)/B_1^{2s+1}(q)$, for functions $se_r(z, q)$

$\sigma_r=\frac{2}{\pi}ce_{2s}(0, q)/A_0^{2s}(q)$ if $p=0$; $\sigma_r=\frac{4}{\pi}ce_{2s+1}(0, q)/A_1^{2s+1}(q)$, if $p=1$; associated with functions $ce_r(z, q)$

$\sigma_r=\frac{4}{\pi}se'_{2s}(0,q)/\sqrt{q}B_2^{2s}(q)$, if $p=0$; $\sigma_r=\frac{2}{\pi}se'_{2s+1}(0, q)/\sqrt{q}B_1^{2s+1}(q)$, if $p=1$; associated with $se_r(z, q)$

Integrals Involving Bessel Function Kernels

Let

20.7.25 $$u=\sqrt{2q(\cosh 2z+\cos 2t)},\ (\mathscr{R}\cosh 2z>1;\text{ if } j=1,\text{ valid also when } z=0)$$

20.7.26

$$Mc_{2r}^{(j)}(z, q)=\frac{(-1)^r 2}{\pi A_0^{2r}}\int_0^{\frac{\pi}{2}}Z_0^{(j)}(u)ce_{2r}(t, q)dt;\ Mc_{2r+1}^{(j)}(z, q)=\frac{(-1)^r 8\sqrt{q}\cosh z}{\pi A_1^{2r+1}}\int_0^{\frac{\pi}{2}}\frac{Z_1^{(j)}(u)\cos t}{u}ce_{2r+1}(t, q)dt$$

20.7.27 $$Ms_{2r}^{(j)}(z, q)=\frac{(-1)^{r+1}8q\sinh 2z}{\pi B_2^{2r}}\int_0^{\frac{\pi}{2}}\frac{Z_2^{(j)}(u)\sin 2t\; se_{2r}(t, q)dt}{u^2}$$

$$Ms_{2r+1}^{(j)}(z, q)=\frac{(-1)^r 8\sqrt{q}\sinh z}{\pi B_1^{2r+1}}\int_0^{\frac{\pi}{2}}\frac{Z_1^{(j)}(u)\sin t\; se_{2r+1}(t, q)dt}{u}$$

In the above the j-convention of **20.4.7** applies and the functions Mc, Ms are defined in **20.5.1–20.5.4**. (These solutions are normalized so that they approach the corresponding Bessel-Hankel functions as $\mathscr{R}z\rightarrow\infty$.)

Other Integrals for $Mc_r^{(1)}(z, q)$ and $Ms_r^{(1)}(z, q)$

20.7.28 $$Mc_r^{(1)}(z, q)=\frac{(-1)^s 2}{\pi ce_r(0, q)}\int_0^{\frac{\pi}{2}}\cos\left(2\sqrt{q}\cosh z\cos t-p\frac{\pi}{2}\right)ce_r(t, q)dt$$

20.7.29 $$Mc_r^{(1)}(z, q)=\tau_r\int_0^{\frac{\pi}{2}}[(1-p)+p\cosh z\cos t]\cos\,(2\sqrt{q}\sinh z\sin t)ce_r(t, q)dt$$

$$r=2s+p,\ p=0, 1;\ \tau_r=\frac{2}{\pi}(-1)^s/ce_{2s}\left(\frac{\pi}{2}, q\right),\text{ if } p=0;\ \tau_r=\frac{2}{\pi}(-1)^{s+1}2\sqrt{q}/ce'_{2s+1}\left(\frac{\pi}{2}, q\right)$$

20.7.30 $$Ms_{2r+1}^{(1)}(z, q)=\frac{2}{\pi}\frac{(-1)^r}{se_{2r+1}\left(\frac{\pi}{2}, q\right)}\int_0^{\frac{\pi}{2}}\sin\,(2\sqrt{q}\sinh z\sin t)se_{2r+1}(t, q)dt$$

20.7.31 $$Ms_{2r+1}^{(1)}(z, q)=\frac{4}{\pi}\frac{\sqrt{q}(-1)^r}{se'_{2r+1}(0, q)}\int_0^{\frac{\pi}{2}}\sinh z\sin t\cos\,(2\sqrt{q}\cosh z\cos t)se_{2r+1}(t, q)dt$$

20.7.32 $$Ms_{2r}^{(1)}(z, q)=\frac{4}{\pi}\sqrt{q}\frac{(-1)^{r+1}}{se'_{2r}(0, q)}\int_0^{\frac{\pi}{2}}\sin\,(2\sqrt{q}\cosh z\cos t)\,[\sinh z\sin t\; se_{2r}(t, q)]dt$$

20.7.33 $$Ms_{2r}^{(1)}(z, q)=\frac{4}{\pi}\frac{(-1)^r\sqrt{q}}{se'_{2r}\left(\frac{\pi}{2}, q\right)}\int_0^{\frac{\pi}{2}}\sin\,(2\sqrt{q}\sinh z\sin t)[\cosh z\cos t\; se_{2r}(t, q)]dt$$