

Other Expressions for the Radial Functions (Valid Over More Limited Regions)

20.6.12
$$Mc_{2r}^{(j)}(z, q) = [ce_{2r}(0, q)]^{-1} \sum_{k=0}^{\infty} (-1)^{k+r} A_{2k}^{2r}(q) Z_{2k}^{(j)}(2\sqrt{q} \cosh z)$$

$$Mc_{2r+1}^{(j)}(z, q) = [ce_{2r+1}(0, q)]^{-1} \sum_{k=0}^{\infty} (-1)^{k+r} A_{2k+1}^{2r+1}(q) Z_{2k+1}^{(j)}(2\sqrt{q} \cosh z)$$

20.6.13
$$Ms_{2r}^{(j)}(z, q) = [se'_{2r}(0, q)]^{-1} \tanh z \sum_{k=1}^{\infty} (-1)^{k+r} 2k B_{2k}^{2r}(q) Z_{2k}^{(j)}(2\sqrt{q} \cosh z)$$

$$Ms_{2r+1}^{(j)}(z, q) = [se'_{2r+1}(0, q)]^{-1} \tanh z \sum_{k=0}^{\infty} (-1)^{k+r} (2k+1) B_{2k+1}^{2r+1}(q) Z_{2k+1}^{(j)}(2\sqrt{q} \cosh z)$$

Valid for $\Re z > 0$, $|\cosh z| > 1$; if $j=1$, valid for all z . They agree with 20.6.7-20.6.10 if the Bessel functions $Y_m(2q^{\frac{1}{2}} \cosh z)$ are made single-valued in a suitable way. For example, let

$$Y_m(u) = \frac{2}{\pi} (\ln u) J_m(u) + \phi(u)$$

where $\phi(u)$ is single-valued for all finite values of u . With $u = 2q^{\frac{1}{2}} \cosh z$, define

20.6.14
$$\ln(2q^{\frac{1}{2}} \cosh z) = \ln 2q^{\frac{1}{2}} + z + \ln \frac{1}{2}(1 + e^{-2z}) \quad -\frac{\pi}{2} \leq \arg \frac{1}{2}(1 + e^{-2z}) \leq \frac{\pi}{2}$$

(If q is not positive, the phase of $\ln 2q^{\frac{1}{2}}$ must also be specified, although this specification will not affect continuity with respect to z . If $Y_m(u)$ is defined from some other expression, the definition must be compatible with 20.6.14.)

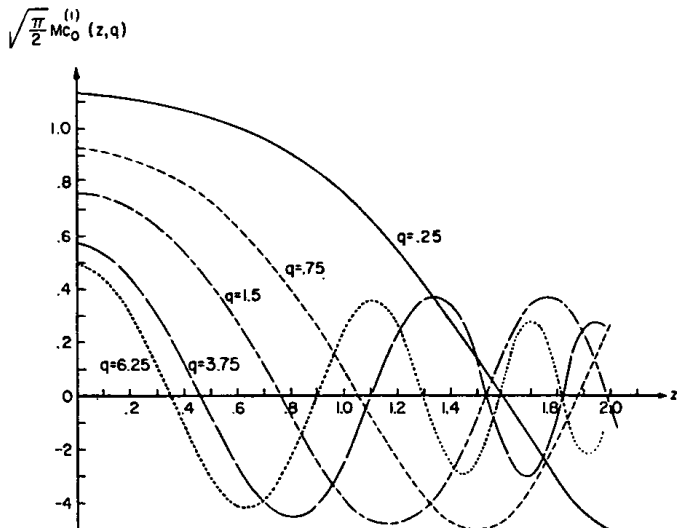


FIGURE 20.11. Radial Mathieu Function of the First Kind. (From J. C. Wiltse and M. J. King, Values of the Mathieu functions, The Johns Hopkins Univ. Radiation Laboratory Tech. Rept. AF-53, 1958, with permission)

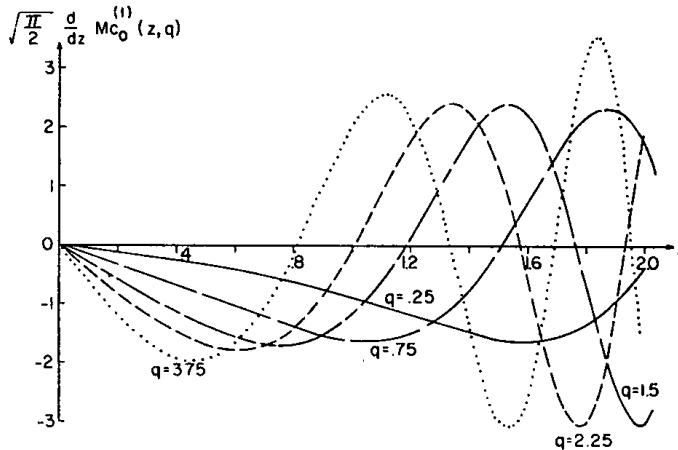


FIGURE 20.12. Derivative of the Radial Mathieu Function of the First Kind. (From J. C. Wiltse and M. J. King, Derivatives, zeros, and other data pertaining to Mathieu functions, The Johns Hopkins Univ. Radiation Laboratory Tech. Rept. AF-57, 1958, with permission)

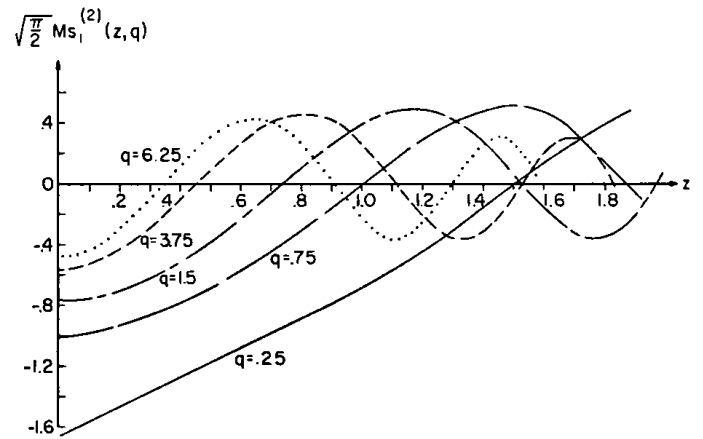


FIGURE 20.13. Radial Mathieu Function of the Second Kind. (From J. C. Wiltse and M. J. King, Values of the Mathieu functions, The Johns Hopkins Univ. Radiation Laboratory Tech. Rept. AF-53, 1958, with permission)

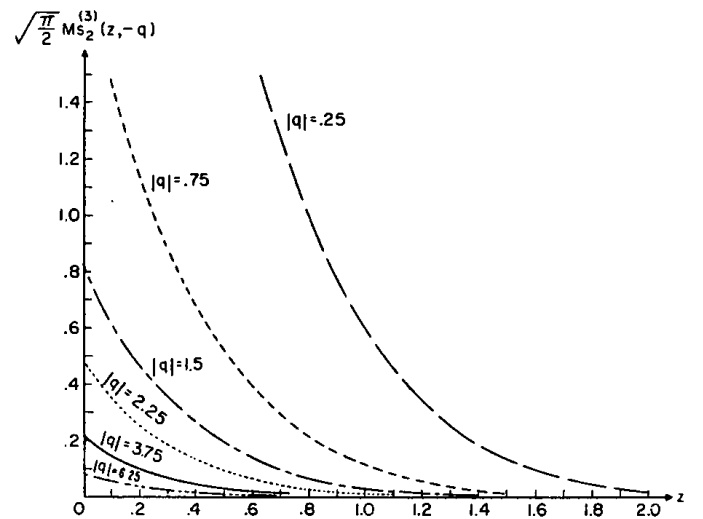


FIGURE 20.14. Radial Mathieu Function of the Third Kind. (From J. C. Wiltse and M. J. King, Values of the Mathieu functions, The Johns Hopkins Univ. Radiation Laboratory Tech. Rept. AF-53, 1958, with permission)