

4.3.35

$$\sin z_1 - \sin z_2 = 2 \cos \left( \frac{z_1 + z_2}{2} \right) \sin \left( \frac{z_1 - z_2}{2} \right)$$

4.3.36

$$\cos z_1 + \cos z_2 = 2 \cos \left( \frac{z_1 + z_2}{2} \right) \cos \left( \frac{z_1 - z_2}{2} \right)$$

4.3.37

$$\cos z_1 - \cos z_2 = -2 \sin \left( \frac{z_1 + z_2}{2} \right) \sin \left( \frac{z_1 - z_2}{2} \right)$$

4.3.38

$$\tan z_1 \pm \tan z_2 = \frac{\sin(z_1 \pm z_2)}{\cos z_1 \cos z_2}$$

4.3.39

$$\cot z_1 \pm \cot z_2 = \frac{\sin(z_2 \pm z_1)}{\sin z_1 \sin z_2}$$

**Relations Between Squares of Sines and Cosines**

4.3.40

$$\sin^2 z_1 - \sin^2 z_2 = \sin(z_1 + z_2) \sin(z_1 - z_2)$$

4.3.41

$$\cos^2 z_1 - \cos^2 z_2 = -\sin(z_1 + z_2) \sin(z_1 - z_2)$$

4.3.42

$$\cos^2 z_1 - \sin^2 z_2 = \cos(z_1 + z_2) \cos(z_1 - z_2)$$

4.3.43

Signs of the Circular Functions in the Four Quadrants

Quadrant	sin csc	cos sec	tan cot
I	+	+	+
II	+	-	-
III	-	-	+
IV	-	+	-

4.3.44

Functions of Angles in Any Quadrant in Terms of Angles in the First Quadrant. ( $0 \leq \theta \leq \frac{\pi}{2}$ ,  $k$  any integer)

	$-\theta$	$\frac{\pi}{2} \pm \theta$	$\pi \pm \theta$	$\frac{3\pi}{2} \pm \theta$	$2k\pi \pm \theta$
sin.....	$-\sin \theta$	$\cos \theta$	$\mp \sin \theta$	$-\cos \theta$	$\pm \sin \theta$
cos.....	$\cos \theta$	$\mp \sin \theta$	$-\cos \theta$	$\pm \sin \theta$	$+\cos \theta$
tan.....	$-\tan \theta$	$\mp \cot \theta$	$\pm \tan \theta$	$\mp \cot \theta$	$\pm \tan \theta$
csc.....	$-\csc \theta$	$+\sec \theta$	$\mp \csc \theta$	$-\sec \theta$	$\pm \csc \theta$
sec.....	$\sec \theta$	$\mp \csc \theta$	$-\sec \theta$	$\pm \csc \theta$	$+\sec \theta$
cot.....	$-\cot \theta$	$\mp \tan \theta$	$\pm \cot \theta$	$\mp \tan \theta$	$\pm \cot \theta$

4.3.45

**Relations Between Circular (or Inverse Circular) Functions**

	$\sin x = a$	$\cos x = a$	$\tan x = a$	$\csc x = a$	$\sec x = a$	$\cot x = a$
sin x.....	$a$	$(1 - a^2)^{\frac{1}{2}}$	$a(1 + a^2)^{-\frac{1}{2}}$	$a^{-1}$	$a^{-1}(a^2 - 1)^{\frac{1}{2}}$	$(1 + a^2)^{-\frac{1}{2}}$
cos x.....	$(1 - a^2)^{\frac{1}{2}}$	$a$	$(1 + a^2)^{-\frac{1}{2}}$	$a^{-1}(a^2 - 1)^{\frac{1}{2}}$	$a^{-1}$	$a(1 + a^2)^{-\frac{1}{2}}$
tan x.....	$a(1 - a^2)^{-\frac{1}{2}}$	$a^{-1}(1 - a^2)^{\frac{1}{2}}$	$a$	$(a^2 - 1)^{-\frac{1}{2}}$	$(a^2 - 1)^{\frac{1}{2}}$	$a^{-1}$
csc x.....	$a^{-1}$	$(1 - a^2)^{-\frac{1}{2}}$	$a^{-1}(1 + a^2)^{\frac{1}{2}}$	$a$	$a(a^2 - 1)^{-\frac{1}{2}}$	$(1 + a^2)^{\frac{1}{2}}$
sec x.....	$(1 - a^2)^{-\frac{1}{2}}$	$a^{-1}$	$(1 + a^2)^{\frac{1}{2}}$	$a(a^2 - 1)^{-\frac{1}{2}}$	$a$	$a^{-1}(1 + a^2)^{\frac{1}{2}}$
cot x.....	$a^{-1}(1 - a^2)^{\frac{1}{2}}$	$a(1 - a^2)^{-\frac{1}{2}}$	$a^{-1}$	$(a^2 - 1)^{\frac{1}{2}}$	$(a^2 - 1)^{-\frac{1}{2}}$	$a$

$\left(0 \leq x \leq \frac{\pi}{2}\right)$  Illustration: If  $\sin x = a$ ,  $\cot x = a^{-1}(1 - a^2)^{\frac{1}{2}}$   
 $\operatorname{arcsec} a = \operatorname{arccot} (a^2 - 1)^{-\frac{1}{2}}$