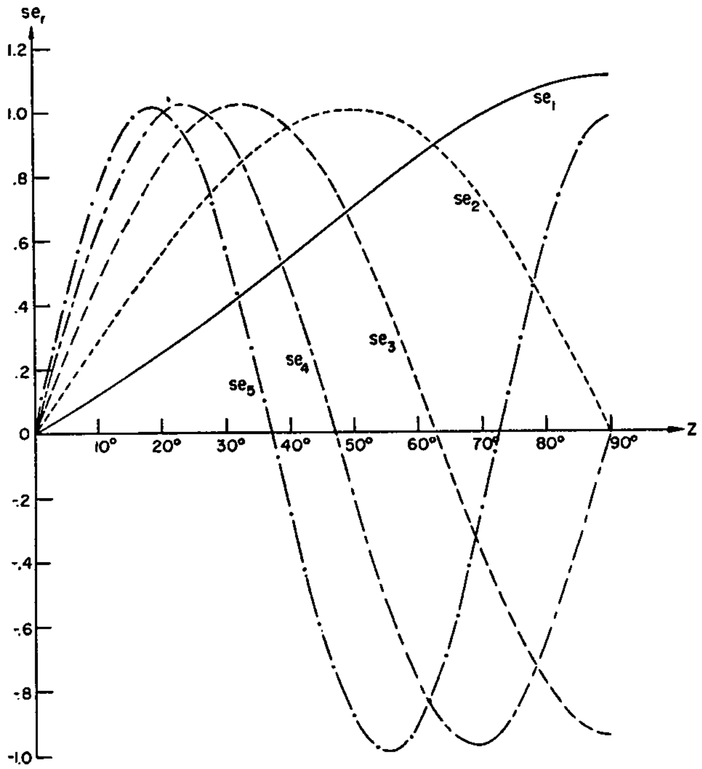

FIGURE 20.3. *Odd Periodic Mathieu Functions, Orders 1–5* $q=1$.

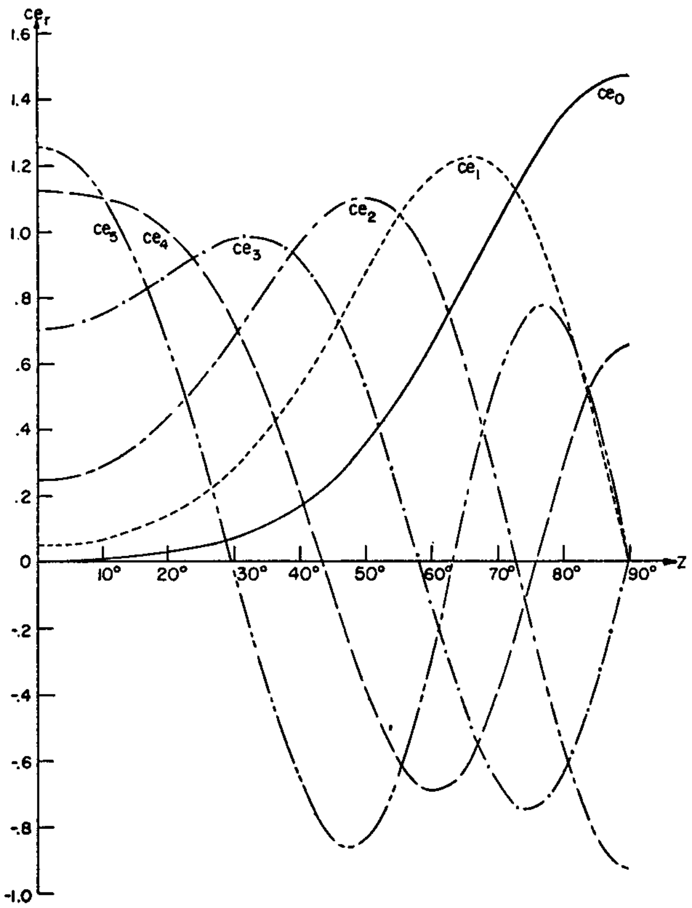

FIGURE 20.4. *Even Periodic Mathieu Functions, Orders 0–5* $q=10$.

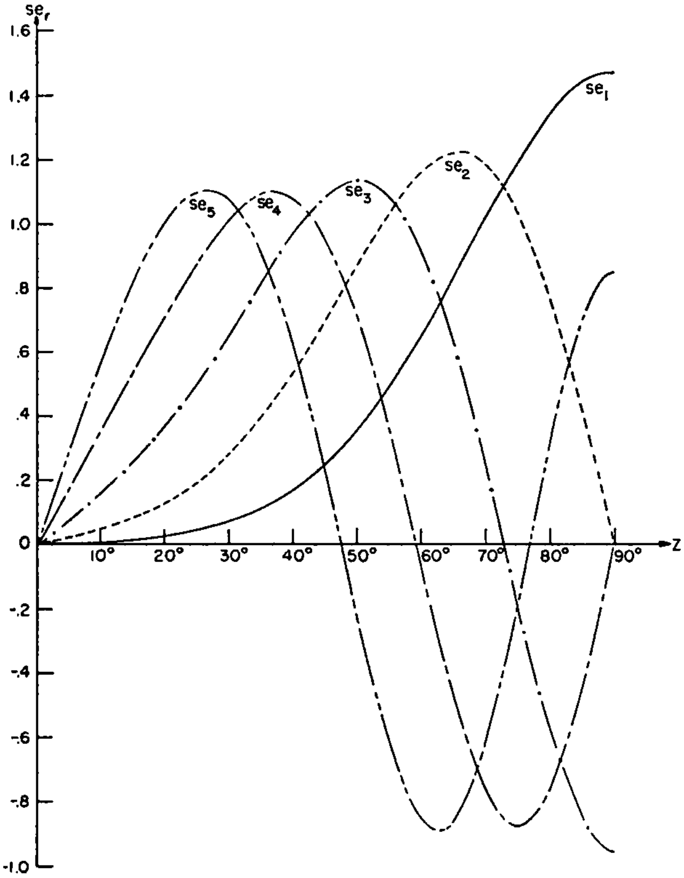

FIGURE 20.5. *Odd Periodic Mathieu Functions, Orders 1–5* $q=10$.

For coefficients associated with above functions

20.2.29

$$A_0^0(0)=2^{-\frac{1}{2}};\ A_r^r(0)=B_r^r(0)=1,\ r>0$$

$$A_{2s}^0=[(-1)^s q^s/s!\,s!\,2^{2s-1}]\,A_0^0+\ .\ .\ .,\ s>0$$

$$\left.\begin{matrix}A^r_{r+2s}\\ B^r_{r+2s}\end{matrix}\right\}=[(-1)^s r!\,q^s/4^s(r+s)!\,s!]\,C_r^r+\ .\ .\ .$$

$$rs>0,\ C_r^r=A_r^r \text{ or } B_r^r$$

$$A^r_{r-2s} \text{ or } B^r_{r-2s}=\frac{(r-s-1)!}{s!(r-1)!}\frac{q^s}{4^s}C_r^r+\ .\ .\ .$$

Asymptotic Expansion for Characteristic Values, $q\gg 1$

Let $w=2r+1$, $q=w^4\varphi$, φ real. Then

20.2.30

$$a_r\sim b_{r+1}\sim -2q+2w\sqrt{q}-\frac{w^2+1}{8}-\frac{\left(w+\frac{3}{w}\right)}{2^7\sqrt{\varphi}}$$

$$-\frac{d_1}{2^{12}\varphi}-\frac{d_2}{2^{17}\varphi^{3/2}}-\frac{d_3}{2^{20}\varphi^2}-\frac{d_4}{2^{25}\varphi^{5/2}}-\ .\ .\ .$$

where

$$d_1=5+\frac{34}{w^2}+\frac{9}{w^4}$$

$$d_2=\frac{33}{w}+\frac{410}{w^3}+\frac{405}{w^5}$$