

$$4.3.3 \quad \tan z = \frac{\sin z}{\cos z}$$

$$4.3.4 \quad \csc z = \frac{1}{\sin z}$$

$$4.3.5 \quad \sec z = \frac{1}{\cos z}$$

$$4.3.6 \quad \cot z = \frac{1}{\tan z}$$

Periodic Properties

$$4.3.7 \quad \sin(z + 2k\pi) = \sin z \quad (k \text{ any integer})$$

$$4.3.8 \quad \cos(z + 2k\pi) = \cos z$$

$$4.3.9 \quad \tan(z + k\pi) = \tan z$$

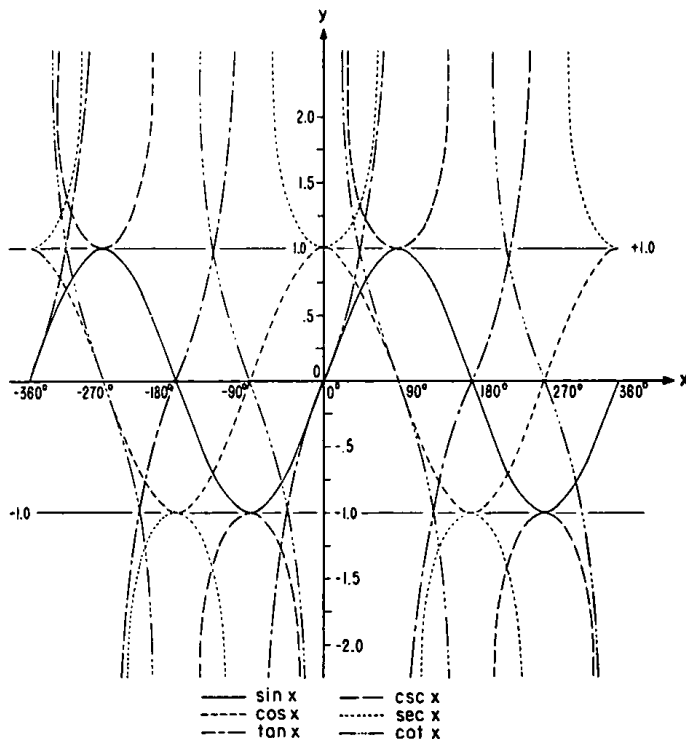


FIGURE 4.3. *Circular functions.*

Relations Between Circular Functions

$$4.3.10 \quad \sin^2 z + \cos^2 z = 1$$

$$4.3.11 \quad \sec^2 z - \tan^2 z = 1$$

$$4.3.12 \quad \csc^2 z - \cot^2 z = 1$$

Negative Angle Formulas

$$4.3.13 \quad \sin(-z) = -\sin z$$

$$4.3.14 \quad \cos(-z) = \cos z$$

$$4.3.15 \quad \tan(-z) = -\tan z$$

Addition Formulas

$$4.3.16 \quad \sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$$

$$4.3.17 \quad \cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$$

$$4.3.18 \quad \tan(z_1 + z_2) = \frac{\tan z_1 + \tan z_2}{1 - \tan z_1 \tan z_2}$$

$$4.3.19 \quad \cot(z_1 + z_2) = \frac{\cot z_1 \cot z_2 - 1}{\cot z_2 + \cot z_1}$$

Half-Angle Formulas

$$4.3.20 \quad \sin \frac{z}{2} = \pm \left(\frac{1 - \cos z}{2} \right)^{\frac{1}{2}}$$

$$4.3.21 \quad \cos \frac{z}{2} = \pm \left(\frac{1 + \cos z}{2} \right)^{\frac{1}{2}}$$

$$4.3.22 \quad \tan \frac{z}{2} = \pm \left(\frac{1 - \cos z}{1 + \cos z} \right)^{\frac{1}{2}} = \frac{1 - \cos z}{\sin z} = \frac{\sin z}{1 + \cos z}$$

The ambiguity in sign may be resolved with the aid of a diagram.

Transformation of Trigonometric Integrals

If $\tan \frac{u}{2} = z$ then

$$4.3.23 \quad \sin u = \frac{2z}{1+z^2}, \quad \cos u = \frac{1-z^2}{1+z^2}, \quad du = \frac{2}{1+z^2} dz$$

Multiple-Angle Formulas

$$4.3.24 \quad \sin 2z = 2 \sin z \cos z = \frac{2 \tan z}{1 + \tan^2 z}$$

$$4.3.25 \quad \cos 2z = 2 \cos^2 z - 1 = 1 - 2 \sin^2 z \\ = \cos^2 z - \sin^2 z = \frac{1 - \tan^2 z}{1 + \tan^2 z}$$

$$4.3.26 \quad \tan 2z = \frac{2 \tan z}{1 - \tan^2 z} = \frac{2 \cot z}{\cot^2 z - 1} = \frac{2}{\cot z - \tan z}$$

$$4.3.27 \quad \sin 3z = 3 \sin z - 4 \sin^3 z$$

$$4.3.28 \quad \cos 3z = -3 \cos z + 4 \cos^3 z$$

$$4.3.29 \quad \sin 4z = 8 \cos^3 z \sin z - 4 \cos z \sin z$$

$$4.3.30 \quad \cos 4z = 8 \cos^4 z - 8 \cos^2 z + 1$$

Products of Sines and Cosines

$$4.3.31 \quad 2 \sin z_1 \sin z_2 = \cos(z_1 - z_2) - \cos(z_1 + z_2)$$

$$4.3.32 \quad 2 \cos z_1 \cos z_2 = \cos(z_1 - z_2) + \cos(z_1 + z_2)$$

$$4.3.33 \quad 2 \sin z_1 \cos z_2 = \sin(z_1 - z_2) + \sin(z_1 + z_2)$$

Addition and Subtraction of Two Circular Functions

4.3.34

$$\sin z_1 + \sin z_2 = 2 \sin \left(\frac{z_1 + z_2}{2} \right) \cos \left(\frac{z_1 - z_2}{2} \right)$$