obtained from differences, for example. We thus obtain $y_1(x_0 \pm H)$ and $y_2(x_0 \pm H)$.

Now suppose

$$W'(a, x_0) = W^{*\prime}(a, x_0) + \lambda$$

then, for all x

$$W(a, x) = y_1(x) + \lambda y_2(x)$$

and in particular

$$W(a, x_0 \pm H) = y_1(x_0 \pm H) + \lambda y_2(x_0 \pm H)$$

The values of $W(a, x_0 \pm H)$ may be read from the tables and two independent estimates of λ obtained, whence

$$W'(a, x_0) = W^{*\prime}(a, x_0) + \lambda$$

to a suitable accuracy.

Example 2. Evaluate $W'(-3, 1)$ using $r=5$. From **Table 19.2**

$W(-3, .5) = -.05857$ $\quad W(-3, 1) = -.61113$

$W(-3, 1.5) = -.69502$

(i) Using the first method

x	$W(-3, x)$	$W''(-3, x)$	δ	δ^2	δ^3
0.4	+0.07298	−0.22186			
0.5	−.05857	+.17937		+131	
0.6	−.18832	.58191			
0.7	−.31226	.97503			
0.8	−.42646	1.34761			
			34081		
0.9	−.52722	1.68842			
			29775		−1095
1.0	−.61113	1.98617			
			24374		−1032
1.1	−.67522	2.22991			
			17941		
1.2	−.71706	2.40932			
1.3	−.73488	2.51513			
1.4	−.72761	2.53936			
1.5	−.69502	2.47601		−9129	
1.6	−.63774	2.32137			

The fifth decimal in $W''(-3, x)$ is only a guard figure which is hardly needed. Only the differences needed have been computed.

Then

$\frac{1}{10} W'(-3, 1)$

$$= \tfrac{1}{10}(-.69502 + .05857) - \frac{1}{1000}(10.38874)$$

$$- \frac{1}{1000}\left\{\tfrac{1}{12}(2.29664) - \frac{1}{240}(-.09260)\right\}$$

$$- \frac{1}{100}\left\{\frac{1}{24}(.54149) - \frac{11}{1440}(-.02127)\right\}$$

$$= -.0636450 - .0103887 - .0001918 - .0002272$$

$$= -.0744527$$

Thus $W'(-3, 1) = -.74453$. This might have an error up to about $1\frac{1}{2}$ units in the last figure but is, in fact, correct to 5 decimals.

(ii) Using the second method, with

$y_1(1) = W(-3, 1) = -.61113$ $\quad$ to 5 decimals

$y_1'(1) = -.745$ $\quad$ to about 3 decimals

the following values result, with $H=.5$,

	y_1	y_2	$W(-3, x) = y_1 + \lambda y_2$
T_0	− .61113	.0000	At $x=1.5$
T_1	− .37250	+.5000	$x - .695223 + .4323\lambda$
			$= -.69502$
T_2	+ .24827 2	.0000	$\lambda = .000203/.4323$
T_3	+ 5680 9	− 677	$= .000470$
T_4	− 1407 4	− 26	So $W'(-3, 1)$
			$= -.745 + \lambda$
			$= -.744530$
T_5	− 279 3	+ 24	At $x=.5$
T_6	+ 13 4	+ 2	$-.058363 - .4371\lambda$
			$= -.05857$
T_7	+ 5 4		$\lambda = .000207/.4371$
T_8	+ 5		$= .000474$
$y(1.5)$	−.695223	+.4323	So $W'(-3, 1)$
$y(.5)$	−.058363	−.4371	$= -.745 + \lambda$
			$= -.744526$

Thus $W'(-3, 1) = -.74453$ which is correct to 5 decimals.

Example 3. Evaluate the positive zero of $U(-3, x)$.

We use **19.7.3** to obtain a first approximation, see **19.26.3.** The appropriate zero of $\mathrm{Ai}(t)$ is at

$$t = (4|a|)^{\frac{2}{3}}\tau = -2.338$$

whence

$$\tau = -(2.338) \times (12)^{-\frac{2}{3}} = -.4461$$

Hence, from **Table 19.3,** $\xi = .3990$ and the approximate zero is $x = 2\sqrt{|a|}\,\xi = 1.382$.

We improve this by using **19.26.10,** but take, for convenience, $x=1.4$ as an approximation, so that the value of U can be read directly from the tables. U' can be obtained as in the section following **Example 1.**

We find

$U(-3, 1.4) = .02627$ $\quad U'(-3, 1.4) = 2.0637$

Then **19.26.9** gives

$u = U/U' = .012730$ $\quad I = -2.51$

$I' = .7$ $\quad I'' = .5$

and