Example 5. $(\Delta>0)$

Given $\omega=10$, $\omega'=55i$, find η, η', $\sigma(\omega)$, $\sigma(\omega')$ and $\sigma(\omega_2)$.

Forming $\omega'/i\omega=5.5$ and entering **Table 18.3** we obtain $\eta=.82246704$, $\sigma(\omega)=.96045\ 40$. Using Legendre's relation we find $\eta'=\eta\omega'-\pi i/2=2.9527\ 723i$. Since interpolation for $\sigma(\omega')$ and $\sigma(\omega+\omega')$ is difficult, use is made of **18.3.15–18.3.17** together with **18.3.4** and **18.3.6.** Values of g_2, g_3 and e_1 can be read directly to eight significant figures and e_3 to about five significant figures giving $g_2=8.1174\ 243$, $g_3=4.4508\ 759$, $e_1=1.6449\ 341$, and $e_3=-.82247$. Use of **18.3.6** yields $H_3=.00174\ 69$ and $H_2=.00174\ 69i$. Application of **18.3.15–18.3.17** yields $\sigma(\omega')/i=.0071177$ and $\sigma(\omega_2)=-.002016-.01055i$. Multiplying the results obtained by the appropriate powers of ω we obtain $\eta=.08224\ 6704$, $\eta'=.29527\ 723i$, $\sigma(\omega)=9.6045\ 40$, $\sigma(\omega')=.071177i$ and $\sigma(\omega_2)=-.02016-.1055i$.

Example 5. $(\Delta<0)$

Given $\omega_2=1000$, $\omega_2'=1004i$, find η_2, η_2', $\sigma(\omega_2)$, $\sigma(\omega_2')$ and $\sigma(\omega')$.

With $\omega_2'/i\omega_2=1.004$, four point interpolation in **Table 18.3** gives $\eta_2=1.5626\ 756$, $\eta_2'=-1.5726\ 664i$, $\sigma(\omega_2)=1.1805\ 028$, $\sigma(\omega_2')=1.1901\ 52i$ and $\sigma(\omega')=.475084+.476717i$.

Multiplying the results obtained by the appropriate powers of ω_2 gives $\eta_2=.00156\ 26756$, $\eta_2'=-.00157\ 26664i$, $\sigma(\omega_2)=1180.5028$, $\sigma(\omega_2')=1190.152i$ and $\sigma(\omega')=475.084+476.717i$.

Determination of Periods from Given Invariants (Table 18.1.)

$\Delta>0$

Given $g_2>0$ and $g_3>0$ such that $\Delta=g_2^3-27g_3^2>0$ (if $g_3=0$, $|\omega'|=\omega$; see lemniscatic case), compute $\bar{g}_2=g_2g_3^{-2/3}$. From **Table 18.1**, determine $\omega g_3^{1/6}$ and $\omega' g_3^{1/6}$, thence ω and ω'.

Example 6.

Given $g_2=10$, $g_3=2$, find ω and ω'. With $\bar{g}_2=g_2g_3^{-2/3}=6.2996\ 05249$, from **Table 18.1** $\omega g_3^{1/6}=1.1267\ 806$ and $\omega' g_3^{1/6}=1.2324\ 295i$ whence $\omega=1.003847$ and $\omega'=1.097970i$.

Example 7.

Given $g_2=8$, $g_3=4$, find ω and ω'. With $\bar{g}_2=g_2g_3^{-2/3}=3.1748\ 02104$, from **Table 18.1** $\omega g_3^{1/6}=1.2718\ 310$ and $\omega' g_3^{1/6}=1.8702\ 425i$ whence $\omega=1.009453$ and $\omega'=1.484413i$.

$\Delta<0$

Given g_2 and $g_3>0$ such that $\Delta=g_2^3-27g_3^2<0$ (if $g_3=0$, $|\omega_2'|=\omega_2$; see pseudo-lemniscatic case), compute $\bar{g}_2=g_2g_3^{-2/3}$. From **Table 18.1**, determine $\omega_2 g_3^{1/6}$ and $\omega_2' g_3^{1/6}$, thence ω_2 and ω_2'.

Example 6.

Given $g_2=-10$, $g_3=2$, find ω_2 and ω_2'. With $\bar{g}_2=g_2g_3^{-2/3}=-10/1.5874\ 0105=-6.2996\ 053$, from **Table 18.1** $\omega_2 g_3^{1/6}=1.5741\ 349$ and $\omega_2' g_3^{1/6}=1.7124\ 396i$ whence $\omega_2=1.40239\ 48$ and $\omega_2'=1.52561\ 02i$.

Example 7.

Given $g_2=7$, $g_3=6$, find ω_2 and ω_2'.
With $\bar{g}_2=g_2g_3^{-2/3}=7/3.3019\ 2725=2.119974$, from **Table 18.1** $\omega_2 g_3^{1/6}=1.3423\ 442$ and $\omega_2' g_3^{1/6}=3.1441\ 141i$ whence $\omega_2=.99579\ 976$ and $\omega_2'=2.33241\ 83i$.

Computation of $\wp$, $\wp'$, or ζ for Given z and Arbitrary g_2, g_3

(or arbitrary periods from which g_2 and g_3 can be computed—
in any case, periods must be known, at least approximately)

First reduce the problem (if necessary) to computation for a point z in the Fundamental Rectangle by use of appropriate results from **18.2.**

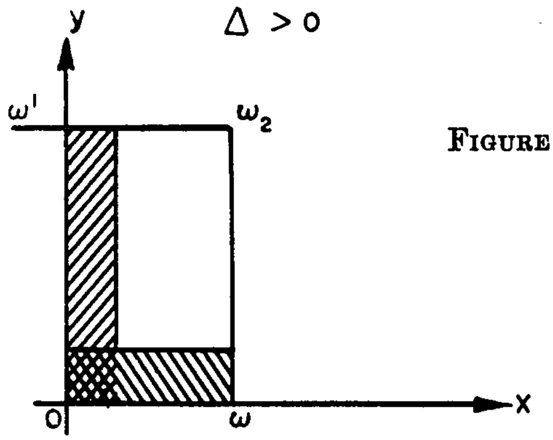

FIGURE 18.16