$\wp'$—Use Laurent's series directly "near" 0 (if $|z|<1$, four terms give at least 8S, five terms at least 11S). Elsewhere, either proceed as for $\wp$ and ζ, or get $\wp'^2=4\wp^3-1$ and extract appropriate square root ($\mathscr{I}\wp'\geqq 0$).

(b) Given $\wp(\wp',\zeta,\sigma)$ corresponding to a point in the Fundamental Triangle, compute z more accurately than can be done with the maps. Only a few significant figures are obtainable from the use of any of the given (truncated) reversed series, except in a small neighborhood of the center of the series. For greater accuracy, use inverse interpolation procedures.

Example 3. Given period ratio a, find parameters m (of elliptic integrals and Jacobi's functions of chapter **16**) and q (of ϑ functions).

m—In both the cases $\Delta>0$ and $\Delta<0$, the period ratio is equal to $K'(m)/K(m)$ (see **18.9**). Knowing K'/K, if $1<K'/K\leqq 3$, use **Table 17.3** to find m; if $K'/K>3$, use the method of **Example 6** in chapter **17.** An alternative method is to use **Table 18.3** to obtain the necessary entries, thence use

$$m=(e_2-e_3)/(e_1-e_3) \text{ in case } \Delta>0,$$

$$m=\tfrac{1}{2}-3e_2/4H_2 \text{ in case } \Delta<0.$$

q—In both the cases $\Delta>0$ and $\Delta<0$, the period ratio determines the exponent for $q[q=e^{-\pi a}$ if $\Delta>0$, $q=ie^{-\pi a/2}$ if $\Delta<0]$. Hence enter **Table 4.16** [$e^{-\pi x}$, $x=0(.01)1$] and multiply the results as appropriate [e.g., $e^{-4.72\pi}=(e^{-\pi})^4(e^{-.72\pi})$].

Determination of Values at Half-Periods, Invariants and Related Quantities from Given Periods (Table 18.3)

$\Delta>0$

Given ω and ω', form $\omega'/i\omega$ and enter **Table 18.3.** Multiply the results obtained by the appropriate power of ω (see footnotes of **Table 18.3**) to obtain value desired.

Example 4.

Given $\omega=10$, $\omega'=11i$, find e_i, g_i, and Δ.

Here $\omega'/i\omega=1.1$, so that direct reading of **Table 18.3** gives

$e_1(1)=1.6843\ 041$

$e_2(1)=-.2166\ 258\ (=-e_1-e_3)$

$e_3(1)=-1.4676\ 783$

$g_2(1)=10.0757\ 7364$

$g_3(1)=2.1420\ 1000.$

Multiplying by appropriate powers of $\omega=10$ we obtain

$e_1=.01684\ 3041$

$e_2=-.00216\ 6258$

$e_3=-.01467\ 6783$

$g_2=1.0075\ 77364 \times 10^{-3}$

$g_3=2.1420\ 1000 \times 10^{-6}$

whence

$\Delta=8.9902\ 3191 \times 10^{-10}$

$\Delta<0$

Given ω_2 and ω_2', form $\omega_2'/i\omega_2$ and enter **Table 18.3.** Multiply the results obtained by the appropriate power of ω_2 (see footnotes of **Table 18.3**) to obtain value desired.

Example 4.

Given $\omega_2=10$, $\omega_2'=11i$, find e_i, g_i, and Δ.

Here $\omega_2'/i\omega_2=1.1$, so that direct reading of **Table 18.3** gives

$e_1(1)=-.2166\ 2576+3.0842\ 589i$

$e_2(1)=.4332\ 5152=-2\mathscr{R}(e_1)$

$e_3(1)=\bar{e}_1(1)$

$g_2(1)=-37.4874\ 912$

$g_3(1)=16.5668\ 099.$

Multiplying by appropriate powers of $\omega_2=10$ we obtain

$e_1=-.00216\ 62576+.03084\ 2589i$

$e_2=.00433\ 25152$

$e_3=\bar{e}_1$

$g_2=-3.7487\ 4912 \times 10^{-3}$

$g_3=1.6566\ 8099 \times 10^{-5}$

whence

$\Delta=-6.0092\ 019 \times 10^{-8}$