Values along $(0, z_0)$

	$\wp$	$\wp'$	ζ	σ
18.14.15 $z_0/4$	$-\frac{i}{2}(\alpha+\sqrt{2\alpha})$	$\alpha(\sqrt{\alpha}+\sqrt{2})e^{i\pi/4}$		$\frac{e^{\pi/64}(2^{1/32})}{\alpha^{1/4}(\sqrt{\alpha}+\sqrt{2})^{1/4}}\,e^{i\pi/4}$
18.14.16 $z_0/2$	$-i/2$	$e^{i\pi/4}$	$\left[\frac{\pi}{4\omega\sqrt{2}}+\frac{1}{2}\right]e^{-i\pi/4}$	$e^{\pi/16}(2^{1/8})e^{i\pi/4}$
18.14.17 $2z_0/3$	$\frac{-i}{2}\sqrt{\sec 30^\circ-1}$	$\frac{e^{i\pi/4}\sqrt[4]{2\sqrt{3}-3}}{\sqrt{3}}$	$\frac{2\eta_2}{3}+\left[\frac{\wp(2z_0/3)}{3}\right]^{1/2}$	$\frac{e^{\pi/9}e^{i\pi/4}(3^{1/6})}{\sqrt[12]{2\sqrt{3}-3}}$
18.14.18 $3z_0/4$	$-\frac{i}{2}(\alpha-\sqrt{2\alpha})$	$\alpha(\sqrt{\alpha}-\sqrt{2})e^{i\pi/4}$		$\frac{e^{9\pi/64}(2^{1/32})}{\alpha^{1/4}(\sqrt{\alpha}-\sqrt{2})^{1/4}}\,e^{i\pi/4}$

$\alpha=1+\sqrt{2}$

Duplication Formulas

18.14.19 $$\wp(2z)=[\wp^2(z)+\tfrac{1}{4}]^2/\{\wp(z)[4\wp^2(z)-1]\}$$

18.14.20
$$\wp'(2z)=(\beta+1)(\beta^2-6\beta+1)/[32\wp'^3(z)],\quad \beta=4\wp^2(z)$$

18.14.21 $$\zeta(2z)=2\zeta(z)+\frac{6\wp^2(z)-\frac{1}{2}}{2\wp'(z)}$$

18.14.22 $$\sigma(2z)=-\wp'(z)\sigma^4(z)$$

Bisection Formulas $(0<x<2\omega)$

18.14.23
$$\wp\left(\frac{x}{2}\right)=[\wp^{\frac{1}{2}}(x)+\{\wp(x)+\tfrac{1}{2}\}^{\frac{1}{2}}]\,[\wp^{\frac{1}{2}}(x)\pm\{\wp(x)-\tfrac{1}{2}\}^{\frac{1}{2}}]$$

[Use + on $0<x\leq\omega$, − on $\omega\leq x<2\omega$]

18.14.24
$$\tfrac{1}{2}\wp'\left(\frac{x}{2}\right)=\wp'(x)\mp[2\wp(x)+\tfrac{1}{2}]\sqrt{\wp(x)-\tfrac{1}{2}}-[2\wp(x)-\tfrac{1}{2}]\sqrt{\wp(x)+\tfrac{1}{2}}-2\wp^{3/2}(x)$$ (See [18.13].)

[Use − on $0<x\leq\omega$, + on $\omega\leq x<2\omega$]

Complex Multiplication

18.14.25 $$\wp(iz)=-\wp(z)$$

18.14.26 $$\wp'(iz)=i\wp'(z)$$

18.14.27 $$\zeta(iz)=-i\zeta(z)$$

18.14.28 $$\sigma(iz)=i\sigma(z)$$

The above equations could be used as follows, e.g.: If z were real, iz would be purely imaginary.

Conformal Maps

Lemniscatic Case

Map: $f(z)=u+iv$

$\wp(z)$

Near zero: $\wp(z)=\frac{1}{z^2}+\epsilon_1$

$$\wp(z)=\frac{1}{z^2}+\frac{z^2}{20}+\epsilon_2,\quad |z|<1$$

Near z_0: $\wp(z)=\frac{-(z-z_0)^2}{4}+\epsilon_3,$

$|z-z_0|<\sqrt{2}$

$$\wp(z)=\frac{-(z-z_0)^2}{4}+\frac{(z-z_0)^6}{80}+\epsilon_4$$