Values along $(0, z_0)$

	$\wp$	$\wp'$	ζ	σ
18.13.26 $z_0/2$	$-2^{1/3}\epsilon^2$	$3i$	$\left[\frac{\eta_2}{\sqrt{3}}+2^{-1/3}\right]e^{-i\pi/6}$	$\frac{e^{\pi/12\sqrt{3}}e^{i\pi/6}}{3^{1/4}}$
18.13.27 $3z_0/4$	$\epsilon^2(e_2-H_2)$	$i(3^{3/4})\sqrt{2-\sqrt{3}}$	$\left[\frac{\pi}{4\omega_2}+\frac{3^{1/4}\sqrt{2-\sqrt{3}}}{2^{4/3}}\right]e^{-i\pi/6}$	$\frac{e^{3\pi/16\sqrt{3}}(2^{1/12})e^{i\pi/6}}{3^{1/4}\sqrt[8]{2-\sqrt{3}}}$
18.13.28 z_0	0	i	$\frac{2\eta_2}{\sqrt{3}}e^{-i\pi/6}$	$e^{\pi/3\sqrt{3}}\cdot e^{i\pi/6}$

Duplication Formulas

18.13.29 $$\wp(2z)=\frac{\wp(z)[\wp^3(z)+2]}{4\wp^3(z)-1}$$

18.13.30 $$\wp'(2z)=\frac{2\wp^6(z)-10\wp^3(z)-1}{[\wp'(z)]^3}$$

18.13.31 $$\zeta(2z)=2\zeta(z)+\frac{3\wp^2(z)}{\wp'(z)}$$

18.13.32 $$\sigma(2z)=-\wp'(z)\sigma^4(z)$$

Trisection Formulas (x real)

18.13.33 $$\wp\left(\frac{x}{3}\right)=\frac{\sqrt[3]{\cos\frac{\phi-\pi}{3}}}{\sqrt[3]{\cos\frac{\phi}{3}}-\sqrt[3]{\cos\frac{\phi+\pi}{3}}}$$

18.13.34 $$\wp'\left(\frac{x}{3}\right)=-\sqrt{3}\,\frac{\sqrt[3]{\cos\frac{\phi}{3}}+\sqrt[3]{\cos\frac{\phi+\pi}{3}}}{\sqrt[3]{\cos\frac{\phi}{3}}-\sqrt[3]{\cos\frac{\phi+\pi}{3}}}$$

where $\tan\phi=\wp'(x)$, $0<x<2\omega_2$ and we must choose ϕ in intervals

$$\left(-\frac{\pi}{2},\frac{\pi}{2}\right), \left(\frac{\pi}{2},\frac{3\pi}{2}\right), \left(\frac{3\pi}{2},\frac{5\pi}{2}\right) \text{ to get}$$

$$\wp\left(\frac{x}{3}\right), \wp\left(\frac{x}{3}+\frac{2\omega_2}{3}\right), \wp\left(\frac{x}{3}+\frac{4\omega_2}{3}\right), \text{ respectively.}$$

Complex Multiplication

18.13.35 $$\wp(\epsilon z)=\epsilon^{-2}\wp(z)$$

18.13.36 $$\wp'(\epsilon z)=-\wp'(z)$$

18.13.37 $$\zeta(\epsilon z)=\epsilon^{-1}\zeta(z)$$

18.13.38 $$\sigma(\epsilon z)=\epsilon\sigma(z)$$

In the above, ϵ denotes (as it does throughout section **18.13**), $e^{i\pi/3}$. The above equations are useful as follows, e.g.:

If z is real, ϵz is on $0\omega'$ **(Figure 18.11)**; if ϵz were purely imaginary, z would be on $0z_0$ **(Figure 18.11)**.

Conformal Maps

Equianharmonic Case

Map: $f(z)=u+iv$

$\wp(z)$

Near zero: $\wp(z)=\frac{1}{z^2}+\epsilon_1$

$$\wp(z)=\frac{1}{z^2}+\frac{z^4}{28}+\epsilon_2$$