Other Series Involving $\wp$

Series near z_0 $[\wp(z_0)=0]$

18.5.52

$$\wp=\wp_0'u\left[1-3c_2u^4-4c_3u^6+\frac{10c_2^2}{3}u^8+\frac{114c_2c_3}{11}u^{10}+\frac{7(12c_3^2-5c_2^3)}{13}u^{12}-\frac{488c_2^2c_3}{33}u^{14}\right]+u^2\left[-5c_2-14c_3u^2+5c_2^2u^4+33c_2c_3u^6+\frac{84c_3^2-10c_2^3}{3}u^8-\frac{1363c_2^2c_3u^{10}}{33}+\frac{5c_2(55c_2^3-2316c_3^2)u^{12}}{143}\right]+\ldots$$

18.5.53 where $u=(z-z_0)$, $\wp_0'\equiv\wp'(z_0)=i\sqrt{g_3}$

18.5.54

$$u=\wp_0'\Big[v+av^2+2a^2v^3+\left(\frac{g_3\wp_0'^2}{2}+5a^3\right)v^4+\frac{a}{5}(3\wp_0'^4+15g_3\wp_0'^2+70a^3)v^5+2a^2(2\wp_0'^4+7g_3\wp_0'^2+21a^3)v^6$$
$$+\left(\frac{g_3\wp_0'^6}{7}+\{g_3^2+20a^3\}\wp_0'^4+15a^2g_3\wp_0'^2+132a^6\right)v^7+15a\left(\frac{g_3\wp_0'^6}{4}+\left\{\frac{3g_3^2}{4}+6a^3\right\}\wp_0'^4+\frac{33ag_3}{2}\wp_0'^2+\frac{143a^6}{5}\right)v^8$$
$$+\frac{5a^2}{2}\left(\frac{2}{3}\wp_0'^8+15g_3\wp_0'^6+\{154a^3+33g_3^2\}\wp_0'^4+\frac{2002a^3g_3\wp_0'^2}{5}+572a^6\right)v^9$$
$$+\frac{1}{4}\left(3\{28a^3+g_3^2\}\wp_0'^8+11g_3\{98a^3+g_3^2\}\wp_0'^6+2002a^3\left\{\frac{16}{5}a^3+g_3^2\right\}\wp_0'^4+16016\,a^6g_3\wp_0'^2+19448\,a^9\right)v^{10}\Big]+\ \ldots$$

18.5.55 where $v=\wp/(\wp_0')^2$ and $a=g_2/4$

Series near ω_i

18.5.56

$$(\wp-e_i)=(3e_i^2-5c_2)u+(10c_2e_i+21c_3)u^2+(7c_2e_i^2+21c_3e_i+5c_2^2)u^3+(18c_3e_i^2+30c_2^2e_i+33c_2c_3)u^4+\left(22c_2^2e_i^2+92c_2c_3e_i+105c_3^2-\frac{10c_2^3}{3}\right)u^5$$
$$+\left(\frac{728}{11}c_2c_3e_i^2+\frac{220}{3}c_2^3e_i+84c_3^2e_i+\frac{1214}{11}c_2^2c_3\right)u^6+\left(\frac{635}{13}c_2^3e_i^2+\frac{855}{13}c_3^2e_i^2+\frac{3405}{11}c_2^2c_3e_i+\frac{45750}{143}c_2c_3^2+\frac{25}{13}c_2^4\right)u^7+\ \ldots,$$

18.5.57 where $u=(z-\omega_i)^2$

Other Series Involving $\wp'$

Series near z_0

18.5.58

$$(\wp'-\wp_0')=\left[-10c_2u-56c_3u^3+30c_2^2u^5+264c_2c_3u^7+\frac{(840c_3^2-100c_2^3)}{3}u^9-\frac{5452c_2^2c_3}{11}u^{11}+\frac{70c_2(55c_2^3-2316c_3^2)}{143}u^{13}\right]$$
$$+\wp_0'\left[-15c_2u^4-28c_3u^6+30c_2^2u^8+114c_2c_3u^{10}+7(12c_3^2-5c_2^3)u^{12}-\frac{2440c_2^2c_3}{11}u^{14}\right]+\ldots$$

18.5.59 where $u=(z-z_0)$

18.5.60

$$(z-z_0)=A-bA^3-\frac{3\wp_0'}{2}A^4+3(c_2+b^2)A^5+10b\wp_0'A^6-3[36c_3-3\wp_0'+4b^3]A^7-3\wp_0'\left(\frac{25}{2}c_2+21b^2\right)A^8+\frac{5}{12}\left(285b^2c_2+100c_2^2-279\wp_0'^2b+132b^4\right)A^9+\ldots$$

18.5.61 where $A=(\wp'-\wp_0')/(-10c_2)$

18.5.62 and $b=4g_3/g_2$

Series near ω_i

18.5.63

$$\wp'=2(3e_i^2-5c_2)\alpha+4(10c_2e_i+21c_3)\alpha^3+6(7c_2e_i^2+21c_3e_i+5c_2^2)\alpha^5+24(6c_3e_i^2+10c_2^2e_i+11c_2c_3)\alpha^7+10\left(22c_2^2e_i^2+92c_2c_3e_i+105c_3^2-\frac{10c_2^3}{3}\right)\alpha^9$$
$$+24\left(\frac{364}{11}c_2c_3e_i^2+\frac{110}{3}c_2^3e_i+42c_3^2e_i+\frac{607}{11}c_2^2c_3\right)\alpha^{11}+70\left(\frac{127}{13}c_2^3e_i^2+\frac{171}{13}c_3^2e_i^2+\frac{681}{11}c_2^2c_3e_i+\frac{9150}{143}c_2c_3^2+\frac{5}{13}c_2^4\right)\alpha^{13}+\ldots,$$

18.5.64 where $\alpha=(z-\omega_i)$.