

$\Delta > 0$ $\Delta < 0$

$$18.3.21 \quad \sigma(\omega/2) = \frac{e^{\eta\omega/8}}{2^{1/4}H_1^{3/8}(2H_1+3e_1)^{1/8}}$$

$$\sigma(\omega_2/2) = \frac{e^{\eta_2\omega_2/8}}{2^{1/4}H_2^{3/8}(2H_2+3e_2)^{1/8}}$$

$$18.3.22 \quad \mathcal{P}(\omega'/2) = e_3 - H_3 < e_3 < 0$$

$$\mathcal{P}(\omega'_2/2) = e_2 - H_2 = \mathcal{P}(\omega_2 + \omega'_2/2) < e_2 < 0$$

$$18.3.23 \quad \mathcal{P}'(\omega'/2) = -2H_3i\sqrt{2H_3-3e_3}$$

$$\mathcal{P}'(\omega'_2/2) = -2H_2i\sqrt{2H_2-3e_2} = \overline{\mathcal{P}'}(\omega_2 + \omega'_2/2)$$

$$18.3.24 \quad \zeta(\omega'/2) = \frac{1}{2}[\eta' - i\sqrt{2H_3-3e_3}]$$

$$\zeta(\omega'_2/2) = \frac{1}{2}[\eta'_2 - i\sqrt{2H_2-3e_2}] = -\zeta(\omega_2 + \omega'_2/2) + 2\eta'$$

$$18.3.25 \quad \sigma(\omega'/2) = \frac{ie^{\eta'\omega'/8}}{2^{1/4}H_3^{3/8}(2H_3-3e_3)^{1/8}}$$

$$\begin{aligned} \sigma(\omega'_2/2) &= \frac{ie^{\eta'_2\omega'_2/8}}{2^{1/4}H_2^{3/8}(2H_2-3e_2)^{1/8}} \\ &= \sigma(\omega_2 + \omega'_2/2) \exp[-\eta'\omega_2] \end{aligned}$$

$$18.3.26 \quad \mathcal{P}(\omega_2/2) = e_2 - H_2$$

$$\mathcal{P}(\omega'/2) = e_3 - H_3$$

$$18.3.27 \quad \mathcal{P}'(\omega_2/2) = -2H_2i(2H_2-3e_2)^{\frac{1}{2}}$$

$$\mathcal{P}'(\omega'/2) = -2iH_3(2H_3-3e_3)^{\frac{1}{2}}$$

$$18.3.28 \quad \zeta(\omega_2/2) = \frac{1}{2}[\eta_2 - i(2H_2-3e_2)^{\frac{1}{2}}]$$

$$\zeta(\omega'/2) = \frac{1}{2}[\eta' - i(2H_3-3e_3)^{\frac{1}{2}}]$$

$$18.3.29 \quad \sigma(\omega_2/2) = \frac{e^{\eta_2\omega_2/8}e^{i\pi/4}}{[4H_2^3(2H_2-3e_2)]^{1/8}}$$

$$\sigma(\omega'/2) = \frac{e^{\eta'\omega'/8}e^{i\pi/4}}{[4H_3^3(2H_3-3e_3)]^{1/8}}$$

One-Third Period Relations

At $z=2\omega_i/3$ ($i=1, 2, 3$) or $2\omega'_i/3$, $\mathcal{P}''^2=12\mathcal{P}\mathcal{P}'^2$;

equivalently:

$$18.3.30 \quad 48\mathcal{P}^4 - 24g_2\mathcal{P}^2 - 48g_3\mathcal{P} - g_2^2 = 0$$

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$$18.3.31 \quad \zeta(2\omega/3) = \frac{2\eta}{3} + \left[\frac{\mathcal{P}(2\omega/3)}{3} \right]^{\frac{1}{2}}$$

$$\zeta(2\omega_2/3) = \frac{2\eta_2}{3} + \left[\frac{\mathcal{P}(2\omega_2/3)}{3} \right]^{\frac{1}{2}}$$

$$18.3.32 \quad \zeta(2\omega'/3) = \frac{2\eta'}{3} - \left[\frac{\mathcal{P}(2\omega'/3)}{3} \right]^{\frac{1}{2}}$$

$$\zeta(2\omega'_2/3) = \frac{2\eta'_2}{3} - \left[\frac{\mathcal{P}(2\omega'_2/3)}{3} \right]^{\frac{1}{2}}$$

$$18.3.33 \quad \zeta(2\omega_2/3) = \frac{2\eta_2}{3} + \left[\frac{\mathcal{P}(2\omega_2/3)}{3} \right]^{\frac{1}{2}}$$

$$\zeta(2\omega'/3) = \frac{2\eta'}{3} + \left[\frac{\mathcal{P}(2\omega'/3)}{3} \right]^{\frac{1}{2}}$$

$$18.3.34 \quad \sigma(2\omega/3) = \frac{-\exp[2\eta\omega/9]}{\sqrt[3]{\mathcal{P}'(2\omega/3)}}$$

$$\sigma(2\omega_2/3) = \frac{-\exp[2\eta_2\omega_2/9]}{\sqrt[3]{\mathcal{P}'(2\omega_2/3)}}$$

$$18.3.35 \quad \sigma(2\omega'/3) = \frac{-\exp[2\eta'\omega'/9]}{[\mathcal{P}'(2\omega'/3)]^{1/3}e^{2\pi i/3}}$$

$$\sigma(2\omega'_2/3) = \frac{-\exp[2\eta'_2\omega'_2/9]}{[\mathcal{P}'(2\omega'_2/3)]^{1/3}e^{2\pi i/3}}$$

$$18.3.36 \quad \sigma(2\omega_2/3) = \frac{-\exp[2\eta_2\omega_2/9]}{[\mathcal{P}'(2\omega_2/3)]^{1/3}e^{2\pi i/3}}$$

$$\sigma(2\omega'/3) = \frac{-\exp[2\eta'\omega'/9]}{[\mathcal{P}'(2\omega'/3)]^{1/3}e^{2\pi i/3}}$$

Legendre's Relation

$$18.3.37 \quad \eta\omega' - \eta'\omega = \pi i/2$$

$$\eta_2\omega'_2 - \eta'_2\omega_2 = \pi i$$

(also valid for $\Delta < 0$)

Relations Among the H_i

$$18.3.38 \quad H_1^2 + H_2^2 + H_3^2 = 3g_2/4$$

$$18.3.39 \quad H_1^2H_2^2 + H_2^2H_3^2 + H_3^2H_1^2 = 0$$