

**Table 17.2 COMPLETE ELLIPTIC INTEGRALS OF THE FIRST AND SECOND KINDS AND THE NOME  $q$  WITH ARGUMENT THE MODULAR ANGLE  $\alpha$**

$$K(\alpha) = \int_0^{\frac{\pi}{2}} (1 - \sin^2 \alpha \sin^2 \theta)^{-\frac{1}{2}} d\theta \quad K'(\alpha) = K(90^\circ - \alpha)$$

$$E(\alpha) = \int_0^{\frac{\pi}{2}} (1 - \sin^2 \alpha \sin^2 \theta)^{\frac{1}{2}} d\theta \quad E'(\alpha) = E(90^\circ - \alpha)$$

$$q(\alpha) = \exp \left[ -\pi K'(\alpha) / K(\alpha) \right] \quad q_1(\alpha) = q(90^\circ - \alpha)$$

$\alpha$	$K(\alpha)$			$K'(\alpha)$			$q(\alpha)$			$90^\circ - \alpha$
$0^\circ$	1.57079	63267	94897	$\infty$			0.00000	00000	00000	$90^\circ$
1	1.57091	59581	27243	5.43490	98296	25564	0.00001	90395	55387	89
2	1.57127	49523	72225	4.74271	72652	78886	0.00007	61698	24680	88
3	1.57187	36105	14009	4.33865	39759	99725	0.00017	14256	42257	87
4	1.57271	24349	95227	4.05275	81695	49437	0.00030	48651	48814	86
5	1.57379	21309	24768	3.83174	19997	84146	0.00047	65699	16867	85
6	1.57511	36077	77251	3.65185	59694	78752	0.00068	66451	27305	84
7	1.57667	79815	92838	3.50042	24991	71838	0.00093	52197	97816	83
8	1.57848	65776	88648	3.36986	80266	68445	0.00122	24470	64294	82
9	1.58054	09338	95721	3.25530	29421	43555	0.00154	85045	16579	81
10	1.58284	28043	38351	3.15338	52518	87839	0.00191	35945	90170	80
11	1.58539	41637	75538	3.06172	86120	38789	0.00231	79450	15821	79
12	1.58819	72125	27520	2.97856	89511	81384	0.00276	18093	29252	78
13	1.59125	43820	13687	2.90256	49406	70027	0.00324	54674	43525	77
14	1.59456	83409	31825	2.83267	25829	18100	0.00376	92262	86978	76
15	1.59814	20021	12540	2.76806	31453	68768	0.00433	34205	09983	75
16	1.60197	85300	86952	2.70806	76145	90486	0.00493	84132	64213	74
17	1.60608	13494	10364	2.65213	80046	30204	0.00558	45970	58517	73
18	1.61045	41537	89663	2.59981	97300	61099	0.00627	23946	95994	72
19	1.61510	09160	67722	2.55073	14496	27254	0.00700	22602	97383	71
20	1.62002	58991	24204	2.50455	00790	01634	0.00777	46804	16442	70
21	1.62523	36677	58843	2.46099	94583	04126	0.00859	01752	53626	69
22	1.63072	91016	30788	2.41984	16537	39137	0.00944	92999	75082	68
23	1.63651	74093	35819	2.38087	01906	04429	0.01035	26461	44729	67
24	1.64260	41437	12491	2.34390	47244	46913	0.01130	08432	78049	66
25	1.64899	52184	78530	2.30878	67981	67196	0.01229	45605	27181	65
26	1.65569	69263	10344	2.27537	64296	11676	0.01333	45085	07947	64
27	1.66271	59584	91370	2.24354	93416	98626	0.01442	14412	80638	63
28	1.67005	94262	69580	2.21319	46949	79374	0.01555	61584	97708	62
29	1.67773	48840	80745	2.18421	32169	49248	0.01673	95077	33023	61
30	1.68575	03548	12596	2.15651	56474	99643	0.01797	23870	08967	60
31	1.69411	43573	05914	2.13002	14383	99325	0.01925	57475	39635	59
32	1.70283	59363	12341	2.10465	76584	91159	0.02059	05967	10437	58
33	1.71192	46951	55678	2.08035	80666	91578	0.02197	80013	16901	57
34	1.72139	08313	74249	2.05706	23227	97365	0.02341	90910	88188	56
35	1.73124	51756	57058	2.03471	53121	85791	0.02491	50625	23981	55
36	1.74149	92344	26774	2.01326	65652	05468	0.02646	71830	76961	54
37	1.75216	52364	68845	1.99266	97557	34209	0.02807	67957	17219	53
38	1.76325	61840	59342	1.97288	22662	74650	0.02974	53239	19583	52
39	1.77478	59091	05608	1.95386	48092	51663	0.03147	42771	20286	51
40	1.78676	91348	85021	1.93558	10960	04722	0.03326	52566	95577	50
41	1.79922	15440	49811	1.91799	75464	36423	0.03511	99625	22096	49
42	1.81215	98536	62126	1.90108	30334	63664	0.03704	02001	87133	48
43	1.82560	18981	35889	1.88480	86573	80404	0.03902	78889	26607	47
44	1.83956	67210	93652	1.86914	75460	26462	0.04108	50703	79885	46
45	1.85407	46773	01372	1.85407	46773	01372	0.04321	39182	63772	45
$90^\circ - \alpha$		$K'(\alpha)$			$K(\alpha)$			$q_1(\alpha)$		$\alpha$

$$\left[ \begin{matrix} (-5)7 \\ 11 \end{matrix} \right]$$

$$\left[ \begin{matrix} (-6)9 \\ 9 \end{matrix} \right]$$

Compiled from G. W. and R. M. Spenceley, Smithsonian elliptic function tables, Smithsonian Miscellaneous Collection, vol. 109, Washington, D.C., 1947 (with permission).