n	a_n	b_n	c_n
0	1.00000 00000	.11111 11111	.99380 79900
1	.55555 55555	.33333 33333	.44444 44444
2	.44444 44444	.43033 14829	.11111 11111
3	.43738 79636	.43733 10380	.00705 64808
4	.43735 95008	.43735 94999	.00002 84628
5	.43735 95003	.43735 95003	0

Thus $K(80/81)=\frac{1}{2}\pi a_5^{-1}=3.59154\ 5001.$

Example 4. Find $E(80/81)$.

First Method

Use **17.3.30** which gives, with $m=80/81$

$$E(80/81)=\frac{10}{9}E(.64)-\frac{1}{5}K(.64)$$
$$=1.01910\ 6047$$

taking $E(.64)$ and $K(.64)$ from **Table 17.1.**

Second Method

Polynomial approximation, **17.3.36** gives $E(80/81)=1.01910\ 60\underline{60}$. The last two figures must be dropped to keep within the limit of accuracy of the method.

Third Method

Arithmetic-geometric mean, **17.6.** The numbers were calculated in **Example 3,** fourth method, and we have

$$\frac{K(80/81)-E(80/81)}{K(80/81)}=\frac{1}{2}[c_0^2+2c_1^2+2^2c_2^2+\ldots+2^5c_5^2]$$
$$=\frac{1}{2}[1.43249\ 71298]$$
$$=.71624\ 85649.$$

Using the value of $K(80/81)$ found in **Example 3,** fourth method, we have

$$E(80/81)=1.01910\ 6048 \text{ to 9D.}$$

Example 5. Find q when $m=.9995.$
Here $m_1=.0005$ and so from **Table 17.4**

$$Q(m)=.06251\ 563013$$
$$q_1=m_1Q(m)=.00003\ 12578\ 15.$$

From **17.3.19**

$$\ln\left(\frac{1}{q}\right)=\pi^2/\ln\left(\frac{1}{q_1}\right)=\pi^2/10.37324\ 1132$$
$$=.95144\ 84701$$
$$q=.38618\ 125.$$

The computation could also be made using common logarithms with the aid of **17.3.20.** The point of this procedure is that it enables us to calculate q_1 without the loss of significant figures which would result from direct interpolation in **Table 17.1.** By this means $\ln(1/q_1)$ can be found without loss of accuracy.

Example 6. Find m to 10D when $K'/K=.25$ and when $K'/K=3.5$.

From **17.3.15** with $K'/K=.25$ we can write the iteration formula

$$m^{(n+1)}=1-16e^{-4\pi}\exp[-\pi L(m^{(n)})/K'(m^{(n)})].$$

Then by iteration using **Tables 17.1** and **17.4**

n	$m^{(n)}$
0	1.
1	.99994 42025
2	.99994 42041
3	.99994 42041

Thus $m=.99994\ 42041.$

From **17.3.16** with $K'/K=3.5$ we can write the iteration formula,

$$m^{(n+1)}=16e^{-3.5\pi}\exp[-\pi L(m_1^{(n)})/K(m^{(n)})]$$

n	$m^{(n)}$
0	0
1	.(3)26841 25043
2	.(3)26837 65
3	.(3)26837 65

Thus $m=.00026\ 83765.$

The above methods in conjunction with the auxiliary **Table 17.4** of $L(m)$ enable us to extend **Table 17.3** for $K'/K>3$, and for $K'/K<.3$.

Example 7. Calculate to 5D the Jacobian elliptic function sn $(.75342|.7)$ using **Table 17.5.**
Here

$$m=\sin^2\alpha=.7,\ \alpha=56.789089°.$$

Thus, sn $(.75342|.7)=\sin\varphi$ where φ is determined from

$$F(\varphi\backslash 56.789089°)=.75342.$$

Inspection of **Table 17.5** shows that φ lies between 40° and 45°. We have from the table of $F(\varphi\backslash\alpha)$