where $\cot^2 \lambda$ is the positive root of the equation $x^2-[\cot^2 \varphi+m \sinh^2\psi \csc^2\varphi-m_1]x-m_1 \cot^2\varphi=0$ and $m \tan^2 \mu=\tan^2\varphi \cot^2\lambda-1$.

17.4.12

$$E(\varphi+i\psi\backslash\alpha)=E(\lambda\backslash\alpha)-iE(\mu\backslash 90^\circ-\alpha)+iF(\mu\backslash 90^\circ-\alpha)+\frac{b_1+ib_2}{b_3}$$

where

$$b_1=\sin^2 \alpha \sin \lambda \cos \lambda \sin^2 \mu(1-\sin^2 \alpha \sin^2 \lambda)^{\frac{1}{2}}$$

$$b_2=(1-\sin^2 \alpha \sin^2 \lambda)(1-\cos^2 \alpha \sin^2 \mu)^{\frac{1}{2}} \sin \mu \cos \mu$$

$$b_3=\cos^2 \mu+\sin^2 \alpha \sin^2 \lambda \sin^2 \mu$$

Amplitude Near to $\pi/2$ (see also 17.5)

If $\cos \alpha \tan \varphi \tan \psi=1$

17.4.13 $F(\varphi\backslash\alpha)+F(\psi\backslash\alpha)=F(\pi/2\backslash\alpha)=K$

17.4.14

$$E(\varphi\backslash\alpha)+E(\psi\backslash\alpha)=E(\pi/2\backslash\alpha)+\sin^2\alpha \sin\varphi \sin\psi$$

Values when φ is near to $\pi/2$ and m is near to unity can be calculated by these formulae.

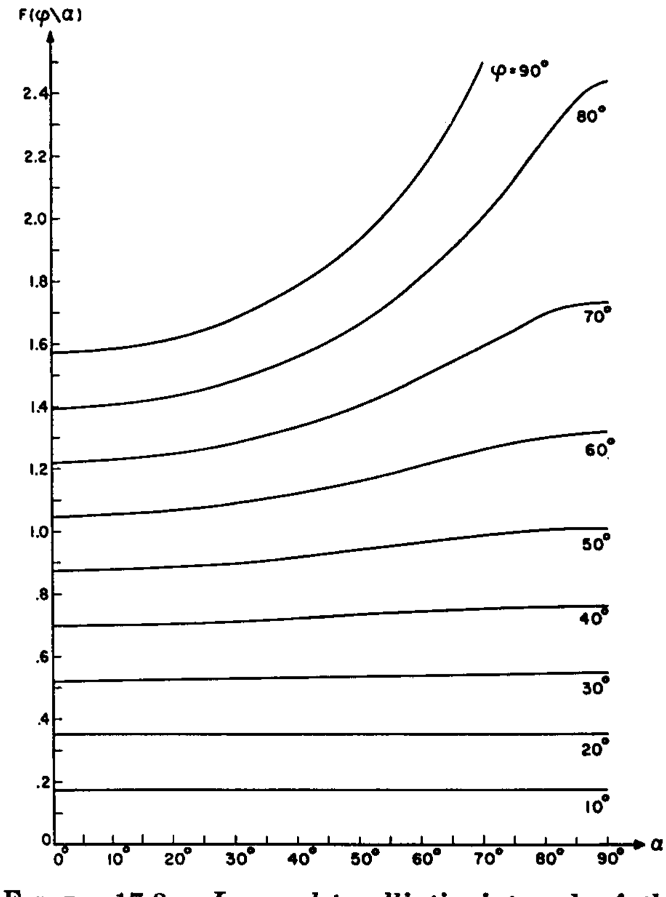

FIGURE 17.3. *Incomplete elliptic integral of the first kind.*

$F(\varphi\backslash\alpha)$, φ constant

Parameter Greater Than Unity

17.4.15 $F(\varphi|m)=m^{-\frac{1}{2}}F(\theta|m^{-1})$, $\sin \theta=m^{\frac{1}{2}} \sin \varphi$

17.4.16 $E(u|m)=m^{\frac{1}{2}}E(um^{\frac{1}{2}}|m^{-1})-(m-1)u$

by which a parameter greater than unity can be replaced by a parameter less than unity.

Negative Parameter

17.4.17

$$F(\varphi|-m)=(1+m)^{-\frac{1}{2}}K(m(1+m)^{-1})-(1+m)^{-\frac{1}{2}}F\left(\frac{\pi}{2}-\varphi \,\middle|\, m(1+m)^{-1}\right)$$

17.4.18

$$E(u|-m)=(1+m)^{\frac{1}{2}}\{E(u(1+m)^{\frac{1}{2}}|m(m+1)^{-1})-m(1+m)^{-\frac{1}{2}}\mathrm{sn}(u(1+m)^{\frac{1}{2}}|m(1+m)^{-1})\,\mathrm{cd}(u(1+m)^{\frac{1}{2}}|m(1+m)^{-1})\}$$

whereby computations can be made for negative parameters, and therefore for pure imaginary modulus.

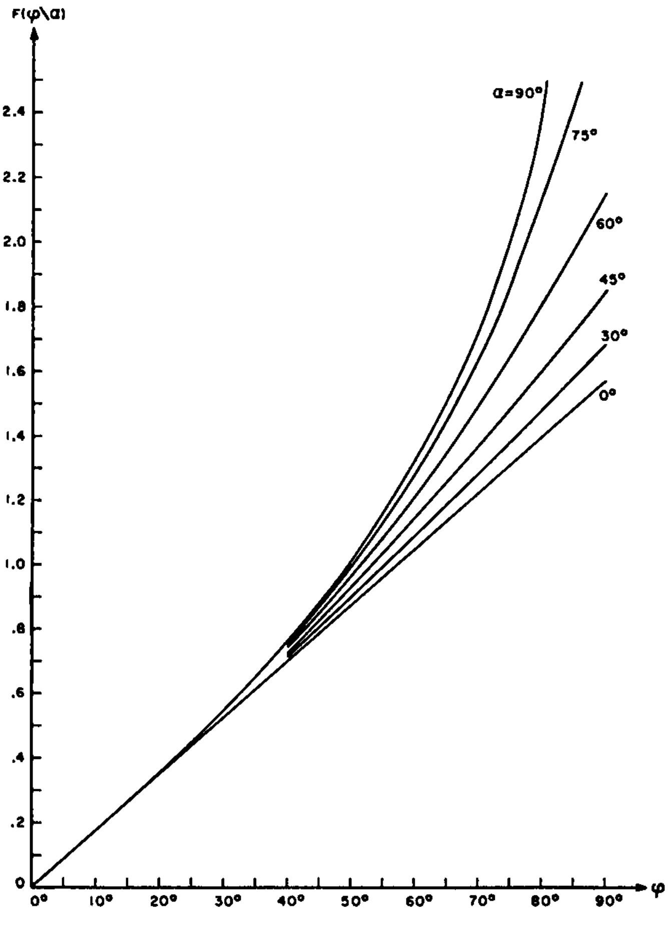

FIGURE 17.4. *Incomplete elliptic integral of the first kind.*

$F(\varphi\backslash\alpha)$, α constant