

Table 16.1

THETA FUNCTIONS

$\epsilon \backslash \alpha$		$\vartheta_s(\epsilon \backslash \alpha)$						$\alpha / \epsilon_1$
$\epsilon \backslash \alpha$	$0^\circ$	$5^\circ$	$10^\circ$	$15^\circ$	$20^\circ$	$25^\circ$	$\alpha / \epsilon_1$	
$0^\circ$	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	90°	
5	0.08715 5743	0.08732 1966	0.08782 4152	0.08867 3070	0.08988 7414	0.09149 5034	85	
10	0.17364 8178	0.17397 9362	0.17497 9967	0.17667 1584	0.17909 1708	0.18229 6223	80	
15	0.25881 9045	0.25931 2677	0.26080 4191	0.26332 6099	0.26693 4892	0.27171 4833	75	
20	0.34202 0143	0.34267 2476	0.34464 3695	0.34797 7361	0.35274 9211	0.35907 2325	70	
25	0.42261 8262	0.42342 4343	0.42586 0446	0.42998 1306	0.43588 2163	0.44370 5382	65	
30	0.50000 0000	0.50095 3708	0.50383 6358	0.50871 3952	0.51570 1435	0.52497 0857	60	
35	0.57357 6436	0.57467 0526	0.57797 7994	0.58357 6134	0.59159 9683	0.60225 0597	55	
40	0.64278 7610	0.64401 3768	0.64772 1085	0.65399 8067	0.66299 9145	0.67495 6130	50	
45	0.70710 6781	0.70845 5688	0.71253 4820	0.71944 3681	0.72935 6053	0.74253 3161	45	
50	0.76604 4443	0.76750 5843	0.77192 5893	0.77941 4712	0.79016 4790	0.80446 5863	40	
55	0.81915 2044	0.82071 4821	0.82544 2256	0.83345 4505	0.84496 1783	0.86028 0899	35	
60	0.86602 5404	0.86767 7668	0.87267 6562	0.88115 1505	0.89332 9083	0.90955 1166	30	
65	0.90630 7787	0.90803 6964	0.91326 9273	0.92214 2410	0.93489 7610	0.95189 9199	25	
70	0.93969 2621	0.94148 5546	0.94691 1395	0.95611 4956	0.96935 0025	0.98700 0216	20	
75	0.96592 5826	0.96776 8848	0.97334 6839	0.98281 0311	0.99642 3213	1.01458 4761	15	
80	0.98480 7753	0.98668 6836	0.99237 4367	1.00202 5068	1.01591 0350	1.03444 0908	10	
85	0.99619 4698	0.99809 5528	1.00384 9133	1.01361 2807	1.02766 2527	1.04641 6011	5	
90	1.00000 0000	1.00190 8098	1.00768 3786	1.01748 5224	1.03158 9925	1.05041 7974	0	
$\epsilon \backslash \alpha$	$30^\circ$	$35^\circ$	$40^\circ$	$45^\circ$	$50^\circ$	$55^\circ$	$\alpha / \epsilon_1$	
$0^\circ$	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	90°	
5	0.09353 4894	0.09606 0073	0.09914 2353	0.10287 9331	0.10740 5819	0.11291 2907	85	
10	0.18636 3367	0.19139 9811	0.19754 9961	0.20501 0420	0.21405 3194	0.22506 4618	80	
15	0.27778 4006	0.28530 3629	0.29449 2321	0.30564 8349	0.31918 5434	0.33569 3043	75	
20	0.36710 5393	0.37706 5455	0.38924 7478	0.40405 4995	0.42204 9614	0.44403 4769	70	
25	0.45365 1078	0.46599 3521	0.48110 6437	0.49950 2749	0.52189 9092	0.54932 5515	65	
30	0.53676 4494	0.55141 5176	0.56937 7735	0.59127 8602	0.61799 6720	0.65080 1843	60	
35	0.61581 3814	0.63268 1725	0.65339 2178	0.67868 8658	0.70961 8904	0.74770 4387	55	
40	0.69019 6708	0.70917 3264	0.73250 7761	0.76106 3101	0.79606 0581	0.83928 2749	50	
45	0.75934 4980	0.78030 3503	0.80611 4729	0.83776 1607	0.87664 1114	0.92480 2089	45	
50	0.82272 9031	0.84552 4503	0.87364 0739	0.90817 9128	0.95071 1025	1.00355 1297	40	
55	0.87986 2121	0.90433 1298	0.93455 6042	0.97175 1955	1.01765 9399	1.07485 2509	35	
60	0.93030 4365	0.95626 6326	0.98837 8598	1.02796 3895	1.07692 1759	1.13807 1621	30	
65	0.97366 6431	1.00092 3589	1.03467 8996	1.07635 2410	1.12798 8100	1.19262 9342	25	
70	1.00961 2870	1.03795 2481	1.07308 5074	1.11651 4503	1.17041 0792	1.23801 2299	20	
75	1.03786 5044	1.06706 1179	1.10328 6100	1.14811 2152	1.20381 2008	1.27378 3626	15	
80	1.05820 3585	1.08801 9556	1.12503 6391	1.17087 7087	1.22789 0346	1.29959 2533	10	
85	1.07047 0366	1.10066 1511	1.13815 8265	1.18461 4727	1.24242 6337	1.31518 2322	5	
90	1.07456 9932	1.10488 6686	1.14254 4218	1.18920 7115	1.24728 6586	1.32039 6454	0	
$\epsilon \backslash \alpha$	$60^\circ$	$65^\circ$	$70^\circ$	$75^\circ$	$80^\circ$	$85^\circ$	$\alpha / \epsilon_1$	
$0^\circ$	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	90°	
5	0.11968 1778	0.12814 8474	0.13904 1489	0.15372 0475	0.17522 3596	0.21321 7690	85	
10	0.23861 4577	0.25558 9564	0.27747 6571	0.30706 5715	0.35063 9262	0.42844 3440	80	
15	0.35604 4091	0.38160 3032	0.41467 2740	0.45960 9511	0.52633 5260	0.64743 4941	75	
20	0.47120 6153	0.50544 4270	0.54994 7578	0.61082 7702	0.70219 9693	0.87146 4767	70	
25	0.58332 3727	0.62633 5361	0.68254 9331	0.76005 8920	0.87783 8622	1.10111 6239	65	
30	0.69160 6043	0.74345 9784	0.81164 3704	0.90647 6281	1.05251 4778	1.33612 3616	60	
35	0.79525 0355	0.85596 1570	0.93630 8263	1.04907 2506	1.22511 1680	1.57526 8297	55	
40	0.89344 6594	0.96294 9380	1.05553 5305	1.18666 0037	1.39412 6403	1.81633 9939	50	
45	0.98538 4972	1.06350 5669	1.16824 3466	1.31788 6740	1.55769 2334	2.05616 7815	45	
50	1.07026 6403	1.15670 0687	1.27329 7730	1.44126 6644	1.71363 1283	2.29072 3417	40	
55	1.14731 5349	1.24161 0747	1.36953 6895	1.55522 4175	1.85953 2258	2.51529 0558	35	
60	1.21579 4546	1.31733 9855	1.45580 7011	1.65814 9352	1.99285 2358	2.72469 4161	30	
65	1.27502 0900	1.38304 3549	1.53099 8883	1.74846 0610	2.11103 3523	2.91357 4159	25	
70	1.32438 1718	1.43795 3601	1.59408 7380	1.82467 1332	2.21162 7685	3.07668 6743	20	
75	1.36335 0417	1.48140 2159	1.64417 0149	1.88545 5864	2.29242 2061	3.20921 2227	15	
80	1.39150 0813	1.51284 3876	1.68050 3336	1.92971 0721	2.35155 6149	3.30704 7313	10	
85	1.40851 9209	1.53187 4716	1.70253 2036	1.95660 6998	2.38762 2438	3.36705 9918	5	
90	1.41421 3562	1.53824 6269	1.70991 3565	1.96563 0511	2.39974 3837	3.38728 7004	0	

$$\sqrt{\sec \alpha} \vartheta_c(\epsilon_1 \backslash \alpha)$$

$$\epsilon^\circ = \frac{\pi}{\lambda} 90^\circ$$

$$\epsilon_1^\circ = 90^\circ - \epsilon^\circ$$

$$\alpha = \arcsin \sqrt{m}$$

$$\vartheta_s(u | m) = \vartheta_s(\epsilon^\circ \backslash \alpha^\circ)$$

In calculating elliptic functions from theta functions, when the modular angle exceeds about 60°, use the descending Landen transformation 16.12 to induce dependence on a smaller modular angle.

Compiled from E. P. Adams and R. L. Hippisley, Smithsonian mathematical formulae and tables of elliptic functions, 3d reprint (The Smithsonian Institution, Washington, D.C., 1957) (with permission).