

**16.4. Calculation of the Jacobian Functions by Use of the Arithmetic-Geometric Mean (A.G.M.)**

For the A.G.M. scale see 17.6.

To calculate  $\text{sn}(u|m)$ ,  $\text{cn}(u|m)$ , and  $\text{dn}(u|m)$  form the A.G.M. scale starting with

16.4.1  $a_0=1, b_0=\sqrt{m_1}, c_0=\sqrt{m},$

terminating at the step  $N$  when  $c_N$  is negligible to the accuracy required. Find  $\varphi_N$  in degrees where

16.4.2 
$$\varphi_N = 2^N a_N u \frac{180^\circ}{\pi}$$

and then compute successively  $\varphi_{N-1}, \varphi_{N-2}, \dots, \varphi_1, \varphi_0$  from the recurrence relation

16.4.3 
$$\sin(2\varphi_{n-1} - \varphi_n) = \frac{c_n}{a_n} \sin \varphi_n.$$

Then

16.4.4

$\text{sn}(u|m) = \sin \varphi_0, \text{cn}(u|m) = \cos \varphi_0$

$$\text{dn}(u|m) = \frac{\cos \varphi_0}{\cos(\varphi_1 - \varphi_0)}$$

From these all the other functions can be determined.

**16.5. Special Arguments**

	$u$	$\text{sn } u$	$\text{cn } u$	$\text{dn } u$
16.5.1	0	0	1	1
16.5.2	$\frac{1}{2}K$	$\frac{1}{(1+m_1^{1/2})^{1/2}}$	$\frac{m_1^{1/4}}{(1+m_1^{1/2})^{1/2}}$	$m_1^{1/4}$
16.5.3	$K$	1	0	$m_1^{1/2}$
16.5.4	$\frac{1}{2}(iK')$	$im^{-1/4}$	$\frac{(1+m^{1/2})^{1/2}}{m^{1/4}}$	$(1+m^{1/2})^{1/2}$
* 16.5.5	$\frac{1}{2}(K+iK')$	$2^{-1/2}m^{-1/4}[(1+m^{1/2})^{1/2} + i(1-m^{1/2})^{1/2}]$	$\left(\frac{m_1}{4m}\right)^{1/4}(1-i)$	$\left(\frac{m_1}{4}\right)^{1/4}[(1+m_1^{1/2})^{1/2} - i(1-m_1^{1/2})^{1/2}]$
16.5.6	$K + \frac{1}{2}(iK')$	$m^{-1/4}$	$-i\left(\frac{1-m^{1/2}}{m^{1/2}}\right)^{1/2}$	$(1-m^{1/2})^{1/2}$
16.5.7	$iK'$	$\infty$	$\infty$	$\infty$
16.5.8	$\frac{1}{2}K + iK'$	$(1-m_1^{1/2})^{-1/2}$	$-i\left(\frac{m_1^{1/2}}{1-m_1^{1/2}}\right)^{1/2}$	$-im_1^{1/4}$
16.5.9	$K + iK'$	$m^{-1/2}$	$-i(m_1/m)^{1/2}$	0

**16.6. Jacobian Functions when  $m=0$  or 1**

		$m=0$	$m=1$
16.6.1	$\text{sn}(u m)$	$\sin u$	$\tanh u$
16.6.2	$\text{cn}(u m)$	$\cos u$	$\text{sech } u$
16.6.3	$\text{dn}(u m)$	1	$\text{sech } u$
16.6.4	$\text{cd}(u m)$	$\cos u$	1
16.6.5	$\text{sd}(u m)$	$\sin u$	$\sinh u$
16.6.6	$\text{nd}(u m)$	1	$\cosh u$
16.6.7	$\text{dc}(u m)$	$\sec u$	1
16.6.8	$\text{nc}(u m)$	$\sec u$	$\cosh u$
16.6.9	$\text{sc}(u m)$	$\tan u$	$\sinh u$
16.6.10	$\text{ns}(u m)$	$\csc u$	$\coth u$
16.6.11	$\text{ds}(u m)$	$\csc u$	$\text{csch } u$
16.6.12	$\text{cs}(u m)$	$\cot u$	$\text{csch } u$
16.6.13	$\text{am}(u m)$	$u$	$\text{gd } u$

\*See page II.